PROCEEDINGS OF THE SYMPOSIUM ON THE FOUNDATIONS OF MATHEMATICS

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Errata

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Page 13, line ↑ 1:

$P \in \Delta_R, \mathrm{car}(P) \simeq p \to m : n, Q \in \Delta_N, \mathrm{car}(Q) \simeq m + n \to k \Rightarrow [p' \to k : n; P; Q] \in \Delta_R$

Page 14, line 2:

$P \in \Delta_R, \mathrm{car}(P) \simeq m : n \to p, Q \in \Delta_N, \mathrm{car}(Q) \simeq m + n \to k \Rightarrow [k : n \to p'; P; Q] \in \Delta_R$

Syntactic Construction of Rational Numbers

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Abstract Syntax

<natnum> = <one>+<natnum><prime> : i,j,k,l,m,n
<positive> = <small>+<medium>+<large> : p,q,r
<small> = <large><star>
<medium> = <one>
<large> = <positive><prime>
<rational> = <negative>+<zero>+<positive> : u,v,w
<negative> = <minus><positive>
<ratio> = <natnum><colon><natnum>
<Nform> = <one>+<Nform><prime>+<Nform><plus><Nform>+<Nform><minus><Nform>
+<Nform><times><Nform> : M,N
<Pform> = <one>+<Pform><prime>+<Pform><star>+<Pform><plus><Pform>
+<Pform><minus><Pform>+<Pform><times><Pform>
+<Pform><slash><Pform> : P,Q
<Qform> = <one>+<zero>+<Qform><prime>+<Qform><star>+<Qform><plus><Qform>
+<Qform><minus><Qform>+<Qform><times><Qform>+<minus><Qform>
+<Qform><slash><Qform> : U,V
<one> := *1*
<prime> := '
<star> := *
<minus> := -
<colon> := :
<times> := •
<slash> := /
<zero> := *0*
<plus> := +

Formal Systems

Δ : P,Q,R,S,T

Δ_N

$[n \to n], [n+1 \to n'], [n'-n \to 1], [n \cdot 1 \to n] \in \Delta_N$

$P \in \Delta_N$, car(P) $\simeq N \to n \Rightarrow [N' \to n'; P] \in \Delta_N$

$P \in \Delta_N$, car(P) $\simeq n+m \to k \Rightarrow [n+m' \to k'; P] \in \Delta_N$

$P \in \Delta_N$, car(P) $\simeq n-m \to k \Rightarrow [n'-m \to k'; P] \in \Delta_N$

$P, Q \in \Delta_N$, car(P) $\simeq n \cdot m \to k$, car(Q) $\simeq k+m \to l \Rightarrow [n \cdot m' \to l; P; Q] \in \Delta_N$

$P, Q, R \in \Delta_N$, car(P) $\simeq N \to n$, car(Q) $\simeq M \to m$, car(R) $\simeq n@m \to k$

$\Rightarrow [N@M \to k; P; Q; R] \in \Delta_N$, whrere $@ \simeq +$ or $-$ or $\cdot$.

Δ_R

$P \in \Delta_N \Rightarrow P \in \Delta_R$

$[1 \to 1:1], [n:n \to 1] \in \Delta_R$

$P \in \Delta_R$, car(P) $\simeq p \to m:n$, $Q \in \Delta_N$, car(Q) $\simeq m+n \to k \Rightarrow [p' \to k:n; P] \in \Delta_R$

$P \in \Delta_R$, $car(P) \simeq p'\to m:n \;\Rightarrow\; [p'^*\to n:m;P] \in \Delta_R$

$P \in \Delta_R$, $car(P) \simeq m:n\to k$, $Q \in \Delta_N$, $car(Q) \simeq m+n\to k \;\Rightarrow\; [k:n\to p';P] \in \Delta_R$

$P \in \Delta_R$, $car(P) \simeq m:n\to p' \;\Rightarrow\; [n:m\to p'^*;P] \in \Delta_R$

Δ_P

$[p\to p] \in \Delta_P$

$P \in \Delta_P$, $car(P) \simeq P\to p \;\Rightarrow\; [P'\to p';P] \in \Delta_P$

$P \in \Delta_P$, $car(P) \simeq P\to p^* \;\Rightarrow\; [P^*\to p;P] \in \Delta_P$

$P \in \Delta_P$, $car(P) \simeq P\to 1 \;\Rightarrow\; [P^*\to 1;P] \in \Delta_P$

$P \in \Delta_P$, $car(P) \simeq P\to p' \;\Rightarrow\; [P^*\to p'^*;P] \in \Delta_P$

$P,Q,T \in \Delta_R$, $R,S \in \Delta_N$, $car(P) \simeq p\to k:l$, $car(Q) \simeq q\to m:n$, $car(R) \simeq k\cdot n \pm l\cdot m\to i$,
$car(S) \simeq l\cdot n\to j$, $car(T) \simeq i:j\to r \;\Rightarrow\; [p\pm q\to r\ ;P;Q;R;S;T] \in \Delta_P$

$P,Q,T \in \Delta_R$, $R,S \in \Delta_N$, $car(P) \simeq p\to k:l$, $car(Q) \simeq q\to m:n$, $car(R) \simeq k\cdot m\to i$,
$car(S) \simeq l\cdot n\to j$, $car(T) \simeq i:j\to r \;\Rightarrow\; [p\cdot q\to r;P;Q;R;S;T] \in \Delta_P$

$P,Q,T \in \Delta_R$, $R,S \in \Delta_N$, $car(P) \simeq p\to k:l$, $car(Q) \simeq q\to m:n$, $car(R) \simeq k\cdot n\to i$,
$car(S) \simeq l\cdot m\to j$, $car(T) \simeq i:j\to r \;\Rightarrow\; [p/q\to r;P;Q;R;S;T] \in \Delta_P$

$P,Q,R \in \Delta_P$, $car(P) \simeq P\to p$, $car(Q) \simeq Q\to q$, $car(R) \simeq p@q\to r$
$\Rightarrow\; [P@Q\to r;P;Q;R] \in \Delta_P$, where $@ \simeq +$ or $-$ or $\cdot$ or $/$.

Δ_Q (Axioms and rules for ', * and - are omitted here.)

$P \in \Delta_P \;\Rightarrow\; P \in \Delta_Q$

$[u\to u], [u+0\to u], [0+u\to u], [p+(-p)\to 0], [(-p)+p\to 0], [u\cdot 0\to 0], [0\cdot u\to 0] \in \Delta_Q$

$P \in \Delta_P$, $car(P) \simeq p-q\to r \;\Rightarrow\; [p+(-q)\to r;P], [(-p)+q\to -r;P] \in \Delta_Q$

$P \in \Delta_P$, $car(P) \simeq q-p\to r \;\Rightarrow\; [p+(-q)\to -r;P], [(-p)+q\to r;P] \in \Delta_Q$

$P \in \Delta_P$, $car(P) \simeq p\cdot q\to r \;\Rightarrow\; [p\cdot(-q)\to -r;P], [(-p)\cdot q\to -r;P], [(-p)\cdot(-q)\to r;P] \in \Delta_Q$

$P,Q,R \in \Delta_Q$, $car(P) \simeq U\to u$, $car(Q) \simeq V\to v$, $car(R) \simeq u@v\to w$
$\Rightarrow\; [U@V\to w;P;Q;R] \in \Delta_Q$, where $@ \simeq +$ or $\cdot$

$P \in \Delta_Q$, $car(P) \simeq U+(-V)\to w \;\Rightarrow\; [U-V\to w;P] \in \Delta_Q$

$P \in \Delta_Q$, $car(P) \simeq U\cdot V^*\to w \;\Rightarrow\; [U/V\to w;P] \in \Delta_Q$.

Notations: $P \vdash U\to u \;\Leftrightarrow\; P \in \Delta_Q,\ car(P) \simeq U\to u$

$\vdash U\to u \;\Leftrightarrow\; \exists P \in \Delta_Q,\ P \vdash U\to u$

$U = V \;\Leftrightarrow\; \exists u(\vdash U\to u$ and $\vdash V\to u)$

Theorem The set <rational> enjoys the usual properties of the rational number field Q.