



Online Quantitative Timed Pattern Matching with Semiring-Valued Weighted Automata

Masaki Waga^{1,2,3}

National Institute of Informatics¹, SOKENDAI²,
JSPS Research Fellow³

27 August 2019, FORMATS 2019

This work is partially supported by JST ERATO HASUO Metamathematics for Systems Design Project (No. JPMJER1603), by JSPS Grants-in-Aid No. 15KT0012 & 18J22498



Monitoring

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Why Monitoring?

Exhaustive formal method

(e.g. model checking, reachability analysis)

- The system is correct/incorrect for any execution
- We need system model (white box)
- Scalability is a big issue

Monitoring

- The system is correct/incorrect for **the given** execution
 - **data-driven** analysis
- We do not need system model (black box is OK)
- Usually scalable

(Qualitative) timed pattern matching

Input

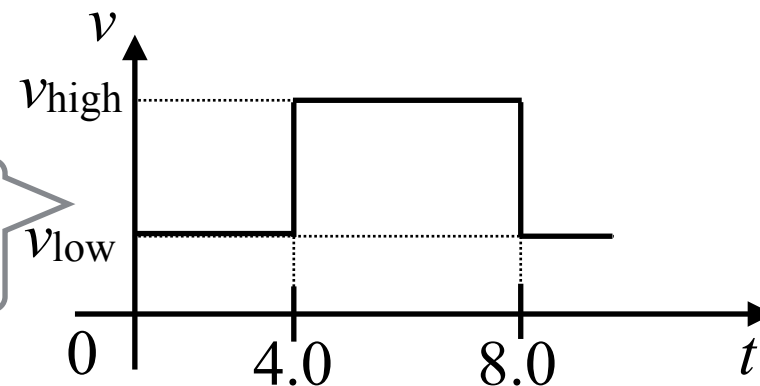
[Ulus+, FORMATS'14]

- **Finite-valued signal** σ

- System **log**

discretized!!

- e.g.,



- **Real-time spec.** \mathcal{W}

- **Spec.** to be monitored

- e.g., The velocity should not keep high for > 1 sec.

Output

- **All** the subsignals $\sigma([t,t'])$ of the **log** satisfies the **spec.**

- e.g., $\sigma([4.0,8.0])$, $\sigma([6.0,8.0])$, $\sigma([6.0,7.5])$, ...

(Qualitative) timed pattern matching

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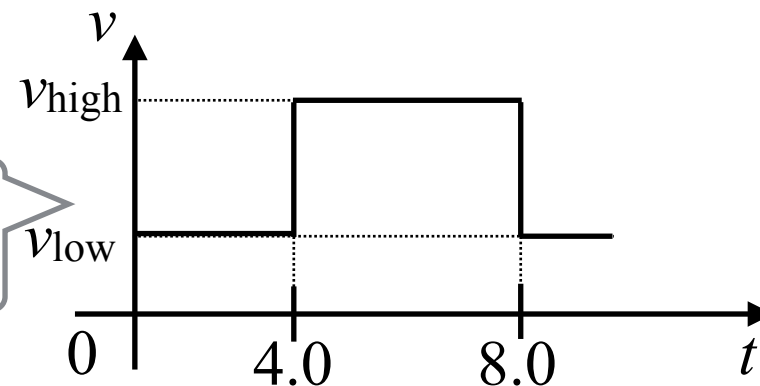
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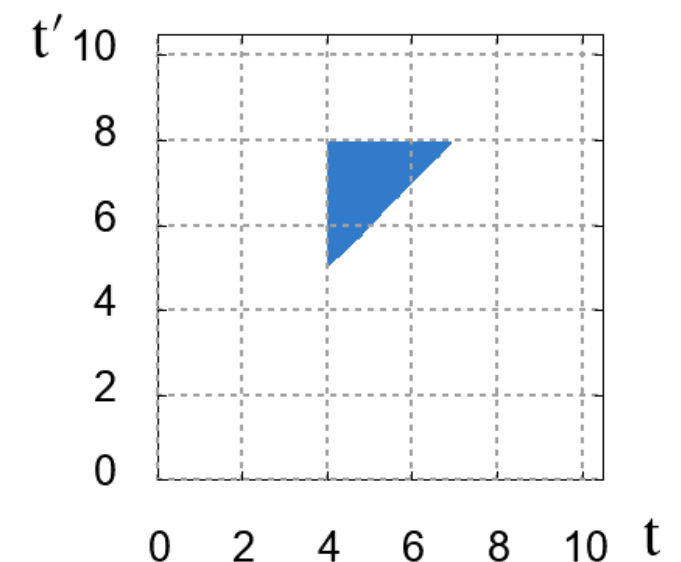
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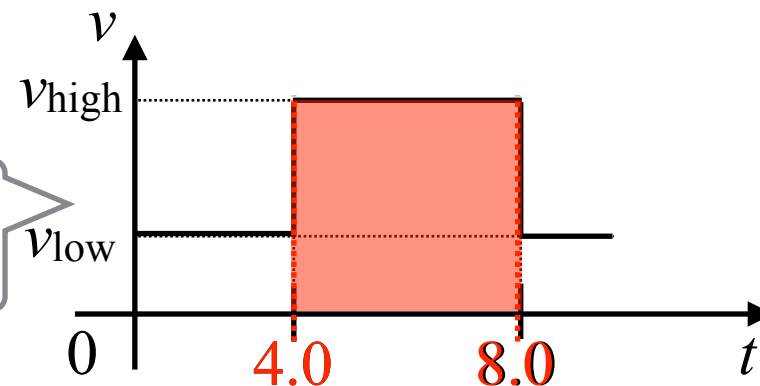
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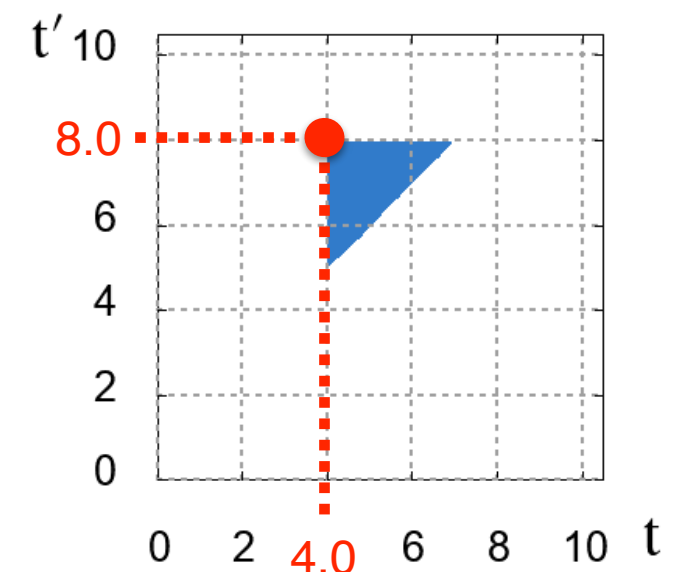
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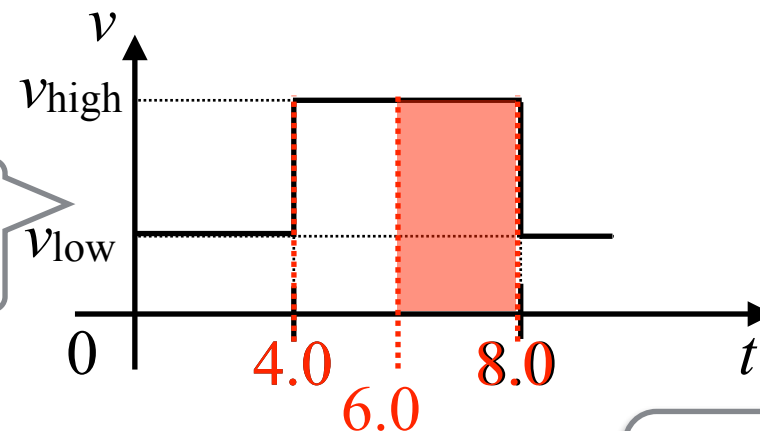
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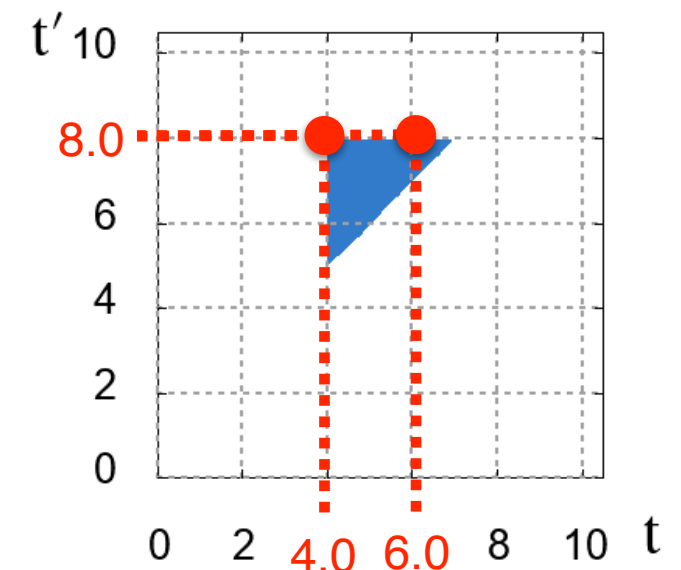
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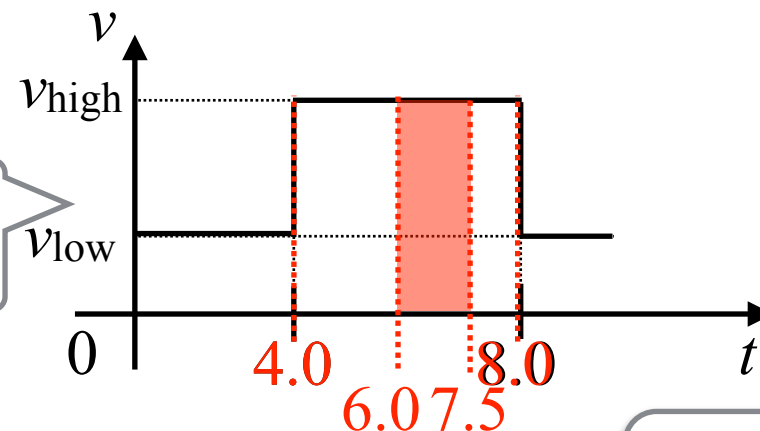
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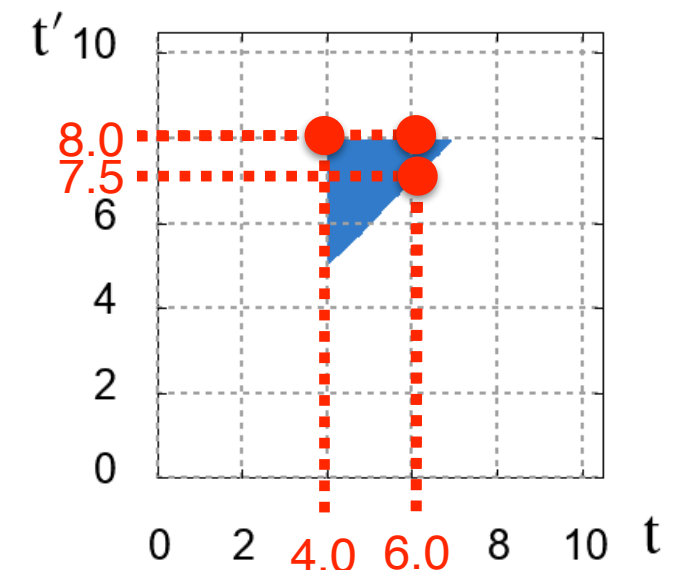
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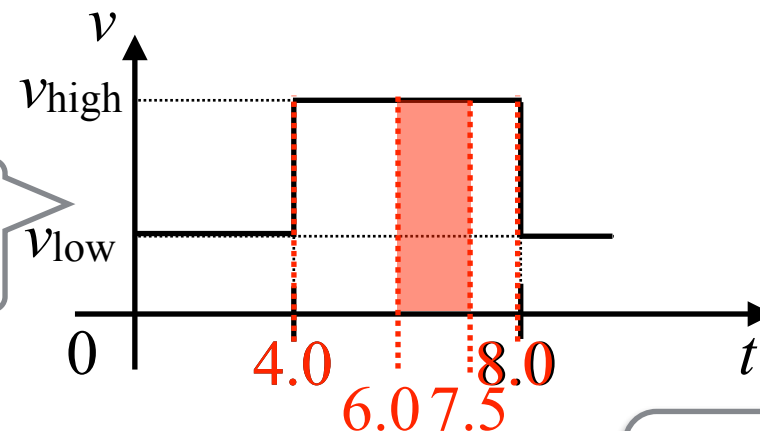
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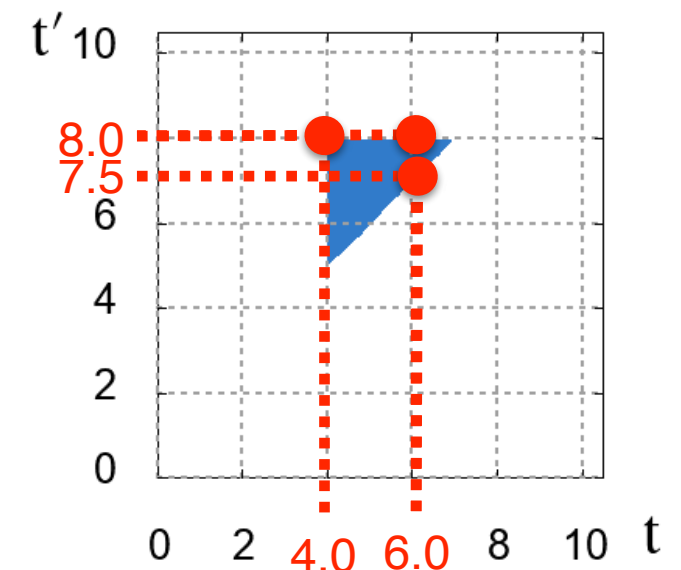
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- e.g., $\sigma([4.0,8.0])$, $\sigma([6.0,8.0])$, $\sigma([6.0,7.5])$, ...

We want to know **how robustly** the spec. is satisfied!!

Quantitative timed pattern matching

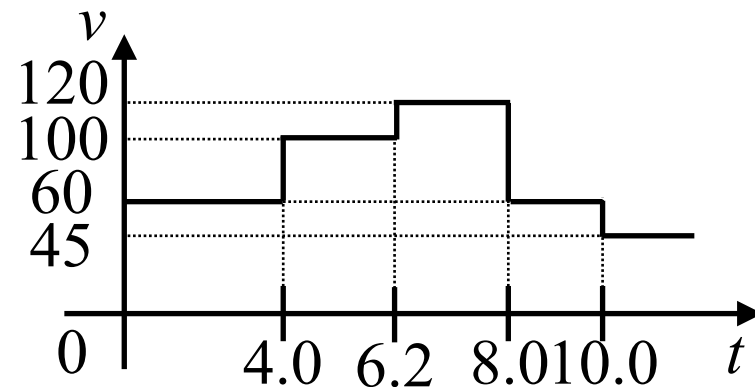
Input

[Bakhirkin+, FORMATS'17]

- **Real-valued piecewise-constant signal σ**

- System **log**

- e.g.,



- **Real-time spec. with signal constraints \mathcal{W}**

- **Spec.** to be monitored

- e.g., The velocity should not keep > 80 for > 1 sec.

Output

- **How robustly**, each subsignals $\sigma([t,t'])$ of the **log**, satisfies the **spec.**

- e.g., $\mathcal{M}(\sigma, \mathcal{W})(2.0,4.0) = -20$, $\mathcal{M}(\sigma, \mathcal{W})(6.5,7.8) = 40$, ...

satisfaction degree of \mathcal{W} for $\sigma([2.0,4.0))$

Quantitative timed pattern matching

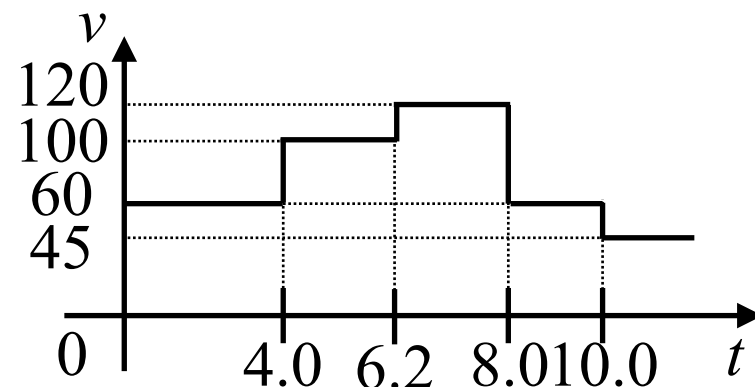
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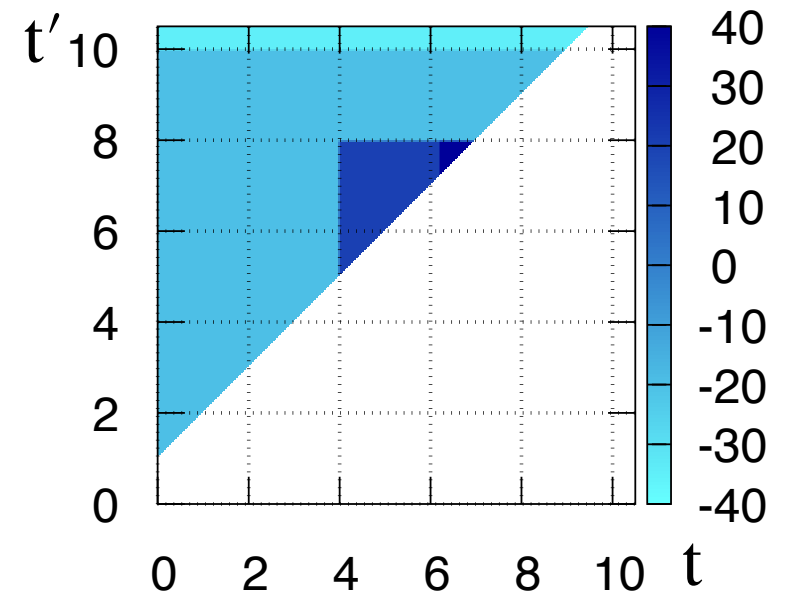
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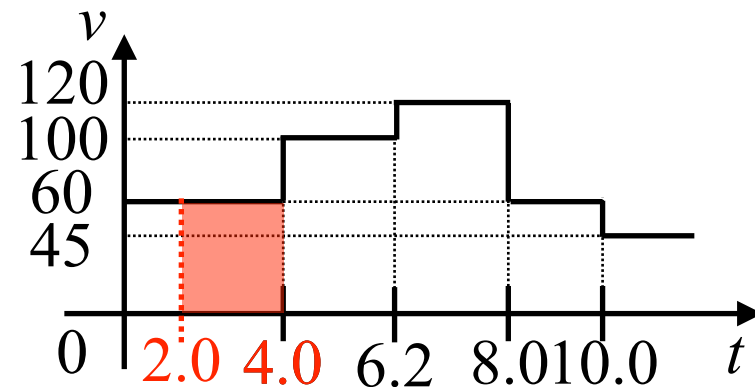
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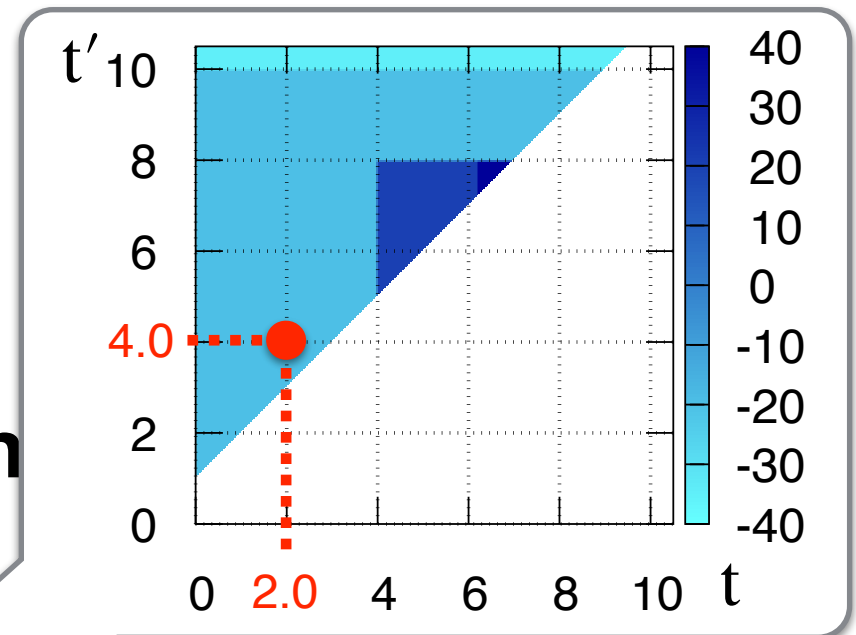
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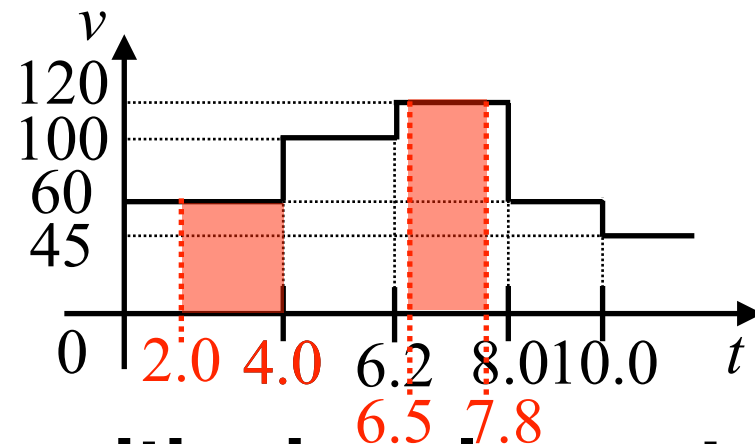
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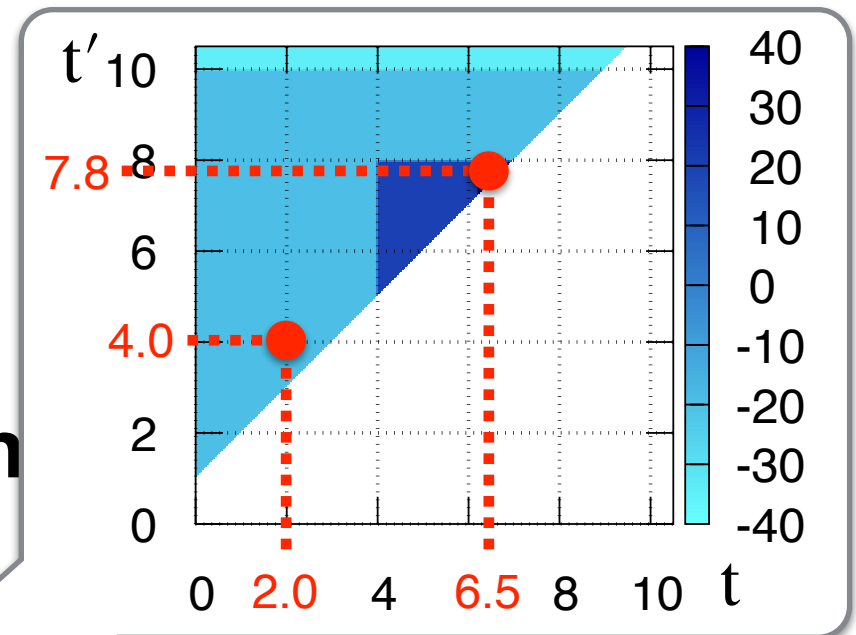
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Output

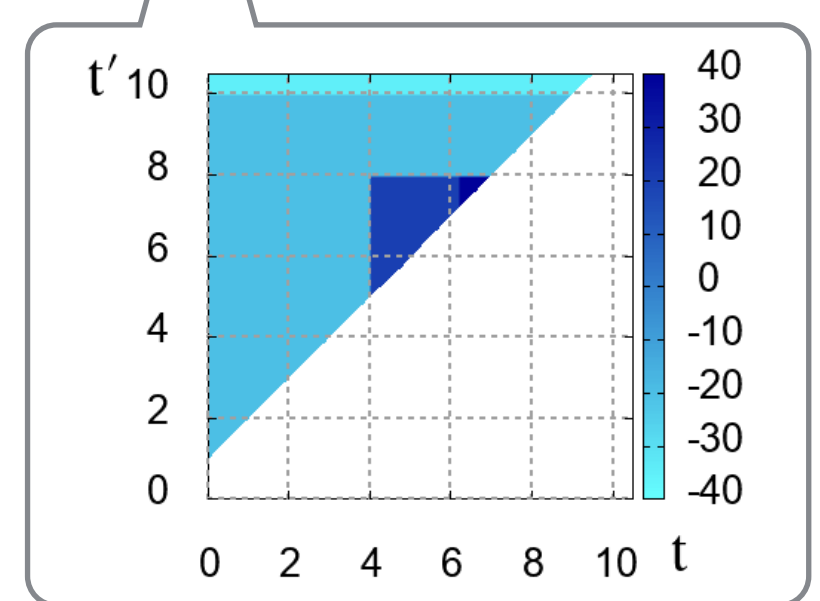
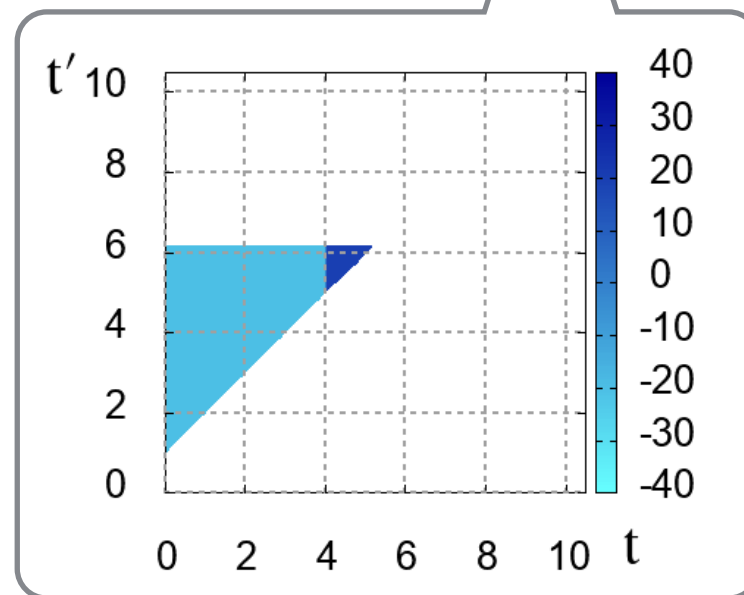
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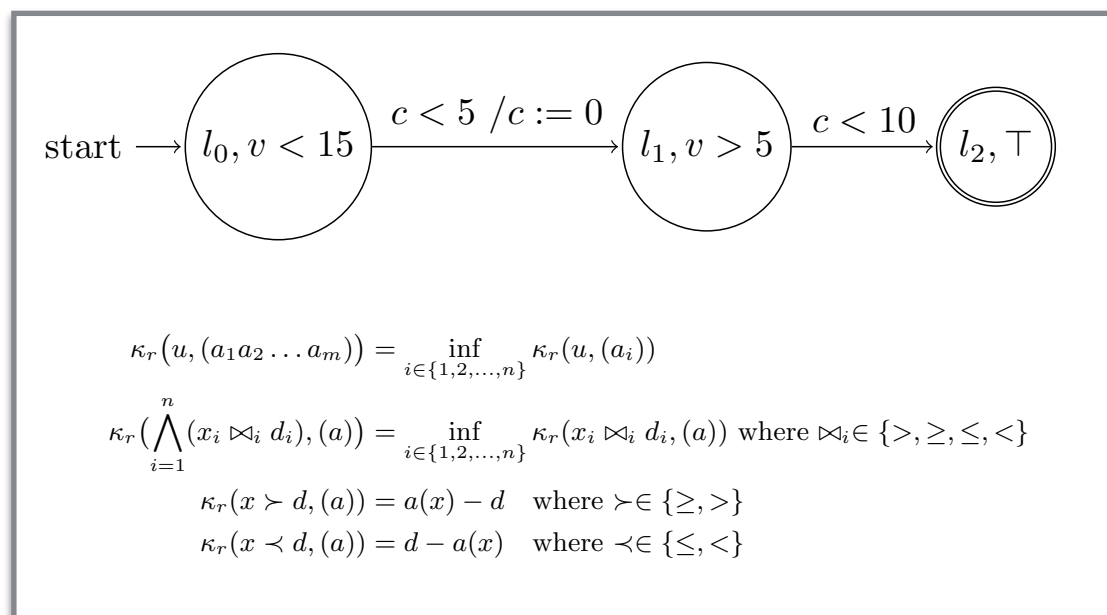
Online Pattern Matching

- After reading the prefix signal σ' of $\sigma = \sigma' \cdot \sigma''$, we obtain the partial result $\mathcal{M}(\sigma', \mathcal{W})$ of $\mathcal{M}(\sigma, \mathcal{W})$
- Important in practice



Timed symbolic weighted automata (TSWA)

- New formalism for spec.
- Automata structure is good for online monitoring
- Generality of semiring (same as the usual WFA)



	Boolean	sup-inf	tropical
\mathcal{S}	{True/False}	$\mathbb{R} \cup \{\pm\infty\}$	$\mathbb{R} \cup \{+\infty\}$
\oplus	\vee	sup	inf
\otimes	\wedge	inf	+

Contribution

- Introduced **timed symbolic weighted automata (TSWA)**
- **TSWA**: timed automata with signal constraints (TSA)
 - **Automata structure**
 - + semiring-valued weight function
 - **Quantitative semantics**
- Gave **online** algorithm for quantitative timed pattern matching
- Implementation + experiments → **Scalable!!**

Related Works

	Qualitative	Quantitative
Offline	[Ulus+, FORMATS'14] (TRE)	[Bakhirkin+, FORMATS'17] (Signal RE)
Online	[Ulus+, TACAS'16], [Bakhirkin+, FORMATS'18] (TRE & TA)	[Contribution] (TSWA)

Only “Robust” Semantics
[Fainekos & Pappas, TCS'09]

↓

Any Semantics defined by semiring-valued weight function

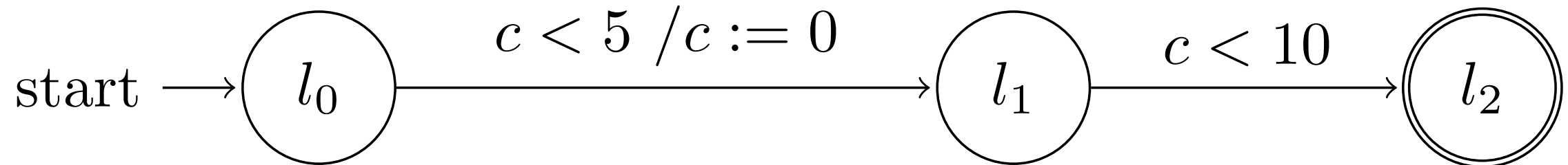
Timed automata → Timed automata with **signal constraints**

Outline

- Motivation + Introduction
- Technical Part
 - Timed symbolic weighted automata (TSWA)
 - TSWA: TA with signal constraints + weight function
 - Quantitative monitoring/timed pattern matching algorithm
 - Idea: zone construction with weight
- Experiments

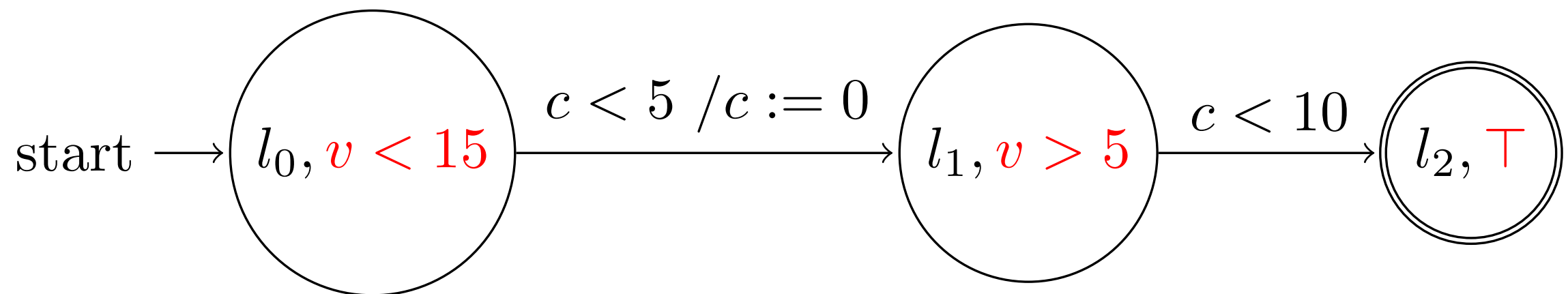
TSWA: TA with signal constraints + weight function

Timed Automaton (TA)



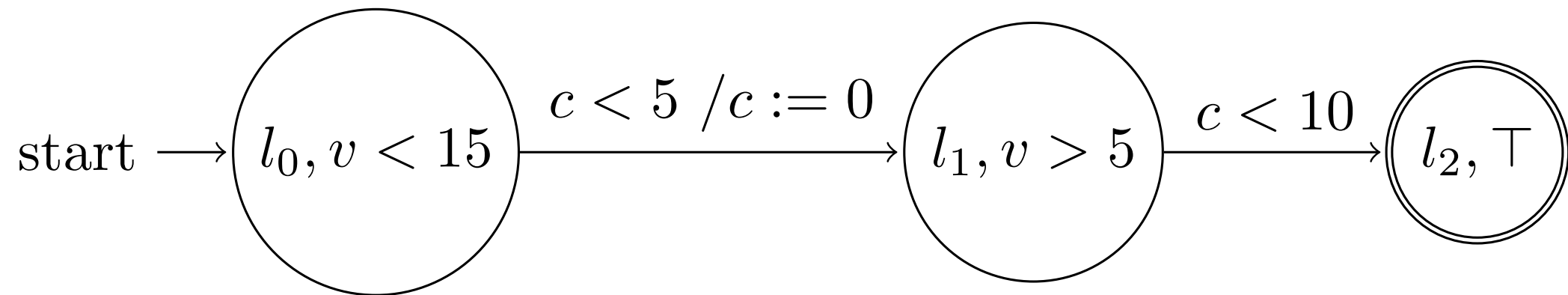
TSWA: TA with signal constraints + weight function

Timed Symbolic Automaton (TSA)



TSWA: TA with signal constraints + weight function

Timed Symbolic Weighted Automaton (TSWA)



+

$$\kappa_r(u, (a_1 a_2 \dots a_m)) = \inf_{i \in \{1, 2, \dots, n\}} \kappa_r(u, (a_i))$$

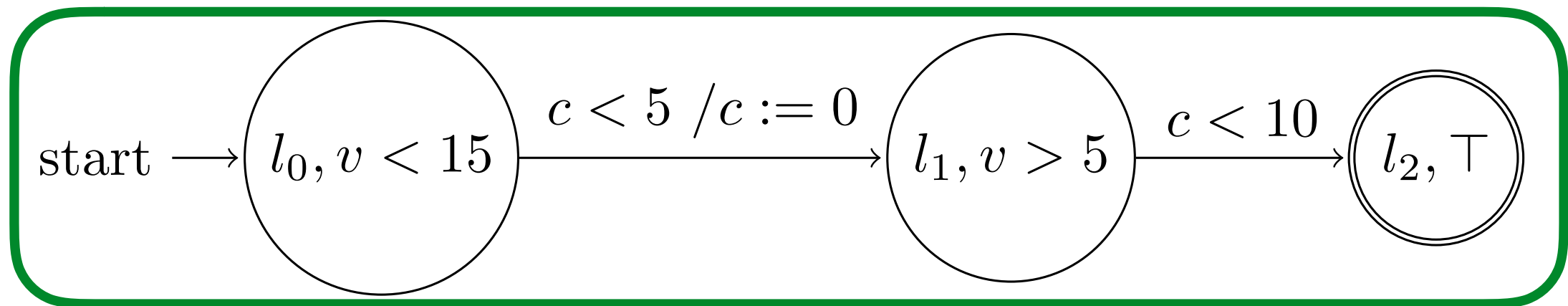
$$\kappa_r\left(\bigwedge_{i=1}^n (x_i \bowtie_i d_i), (a)\right) = \inf_{i \in \{1, 2, \dots, n\}} \kappa_r(x_i \bowtie_i d_i, (a)) \text{ where } \bowtie_i \in \{>, \geq, \leq, <\}$$

$$\kappa_r(x \succ d, (a)) = a(x) - d \text{ where } \succ \in \{\geq, >\}$$

$$\kappa_r(x \prec d, (a)) = d - a(x) \text{ where } \prec \in \{\leq, <\}$$

TSWA: TA with signal constraints + weight function

Timed Symbolic Weighted Automaton (TSWA)



Automata structure

+

Quantitative semantics

$$\kappa_r(u, (a_1 a_2 \dots a_m)) = \inf_{i \in \{1, 2, \dots, m\}} \kappa_r(u, (a_i))$$

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Weight function

$$\kappa: \Phi(X, \mathbb{D}) \times (\mathbb{D}^X)^{\otimes} \rightarrow \mathcal{S}$$

Constraints on signal values at the location

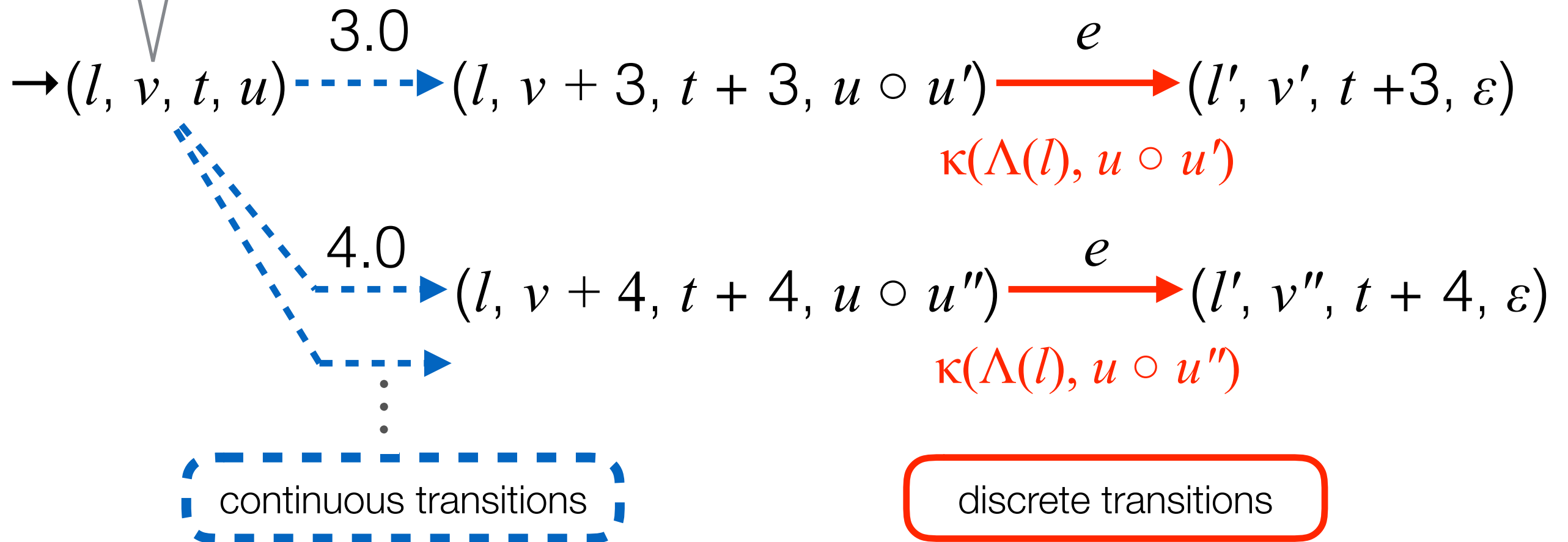
Sequence of signal values at the location

Semiring value

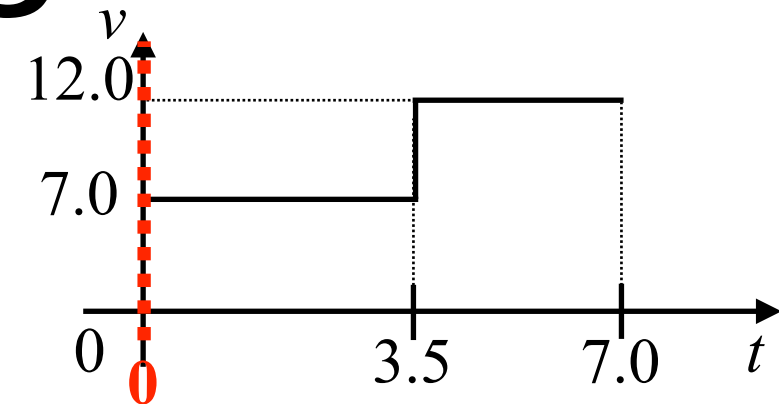
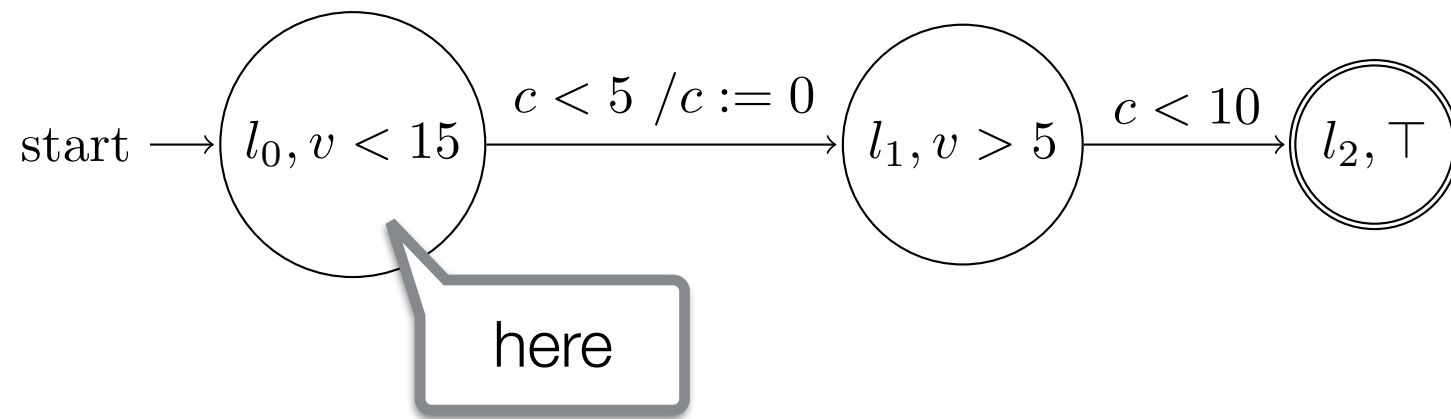
- $\kappa(\Lambda(l), u)$: weight for the stay at l with signal values u
- Semiring: set \mathcal{S} with accumulating operators \oplus and \otimes
- We can use any complete and idempotent semiring

Semantics: Weighted TTS

- l : location
- v : clock valuation
- t : absolute time
- u : sequence of signal values after the latest discrete transition

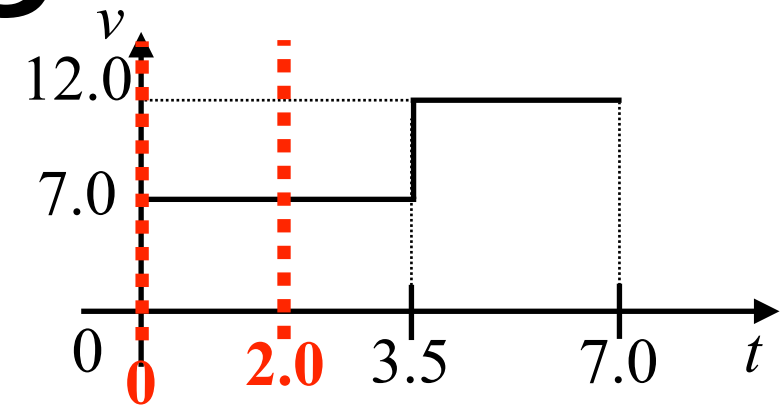
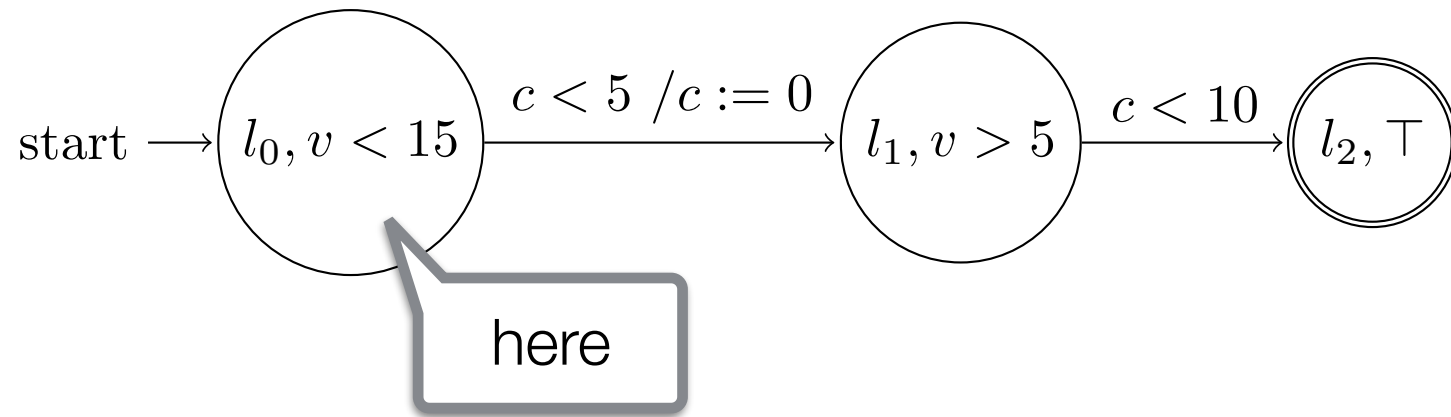


One path in Weighted TTS



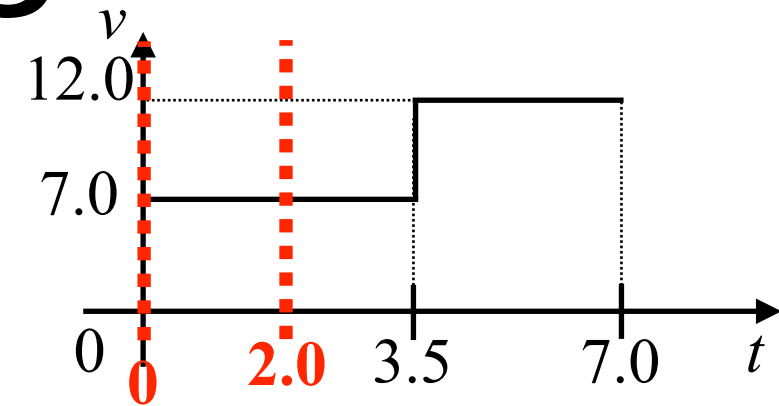
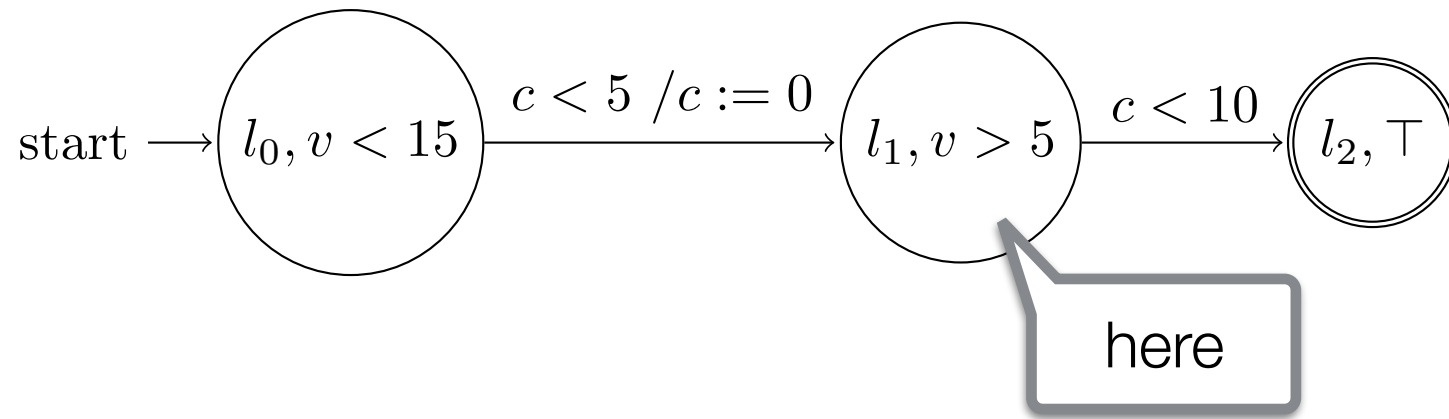
$\rightarrow (l_0, c=0, 0, \varepsilon)$

One path in Weighted TTS



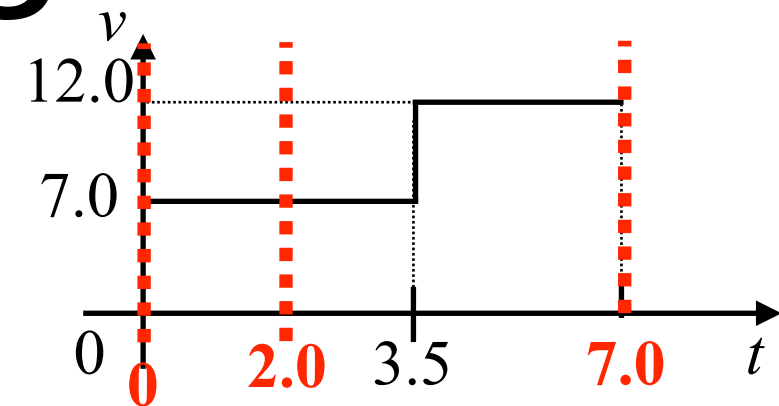
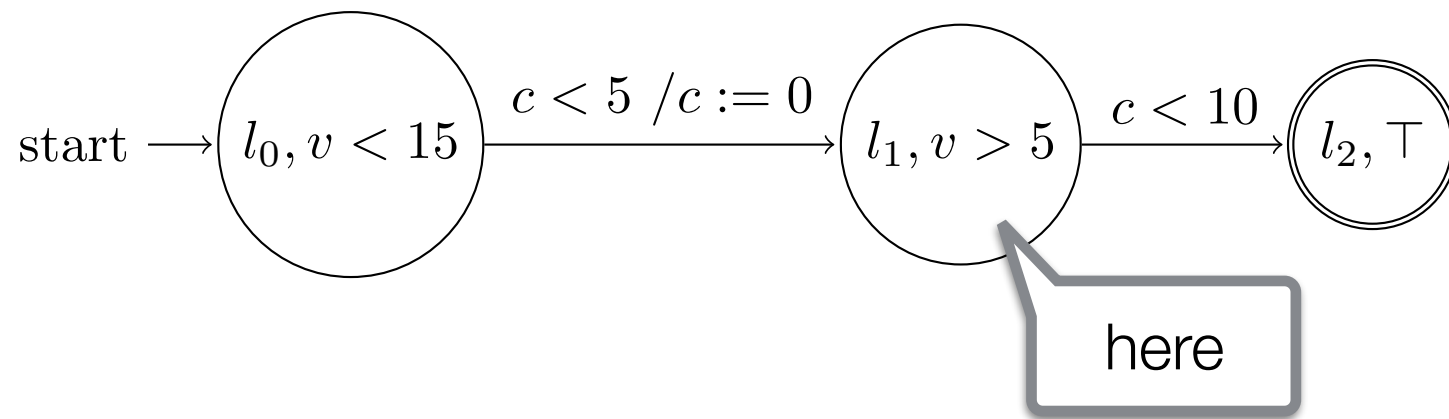
$$\rightarrow (l_0, c=0, 0, \varepsilon) \xrightarrow{2.0} (l_0, c=2, 2, \{v = 7\})$$

One path in Weighted TTS



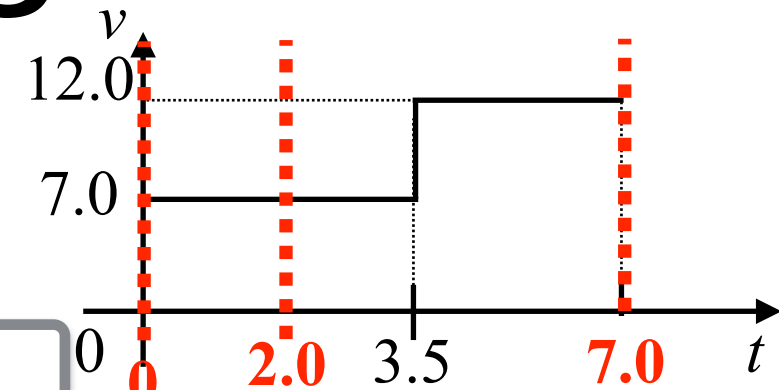
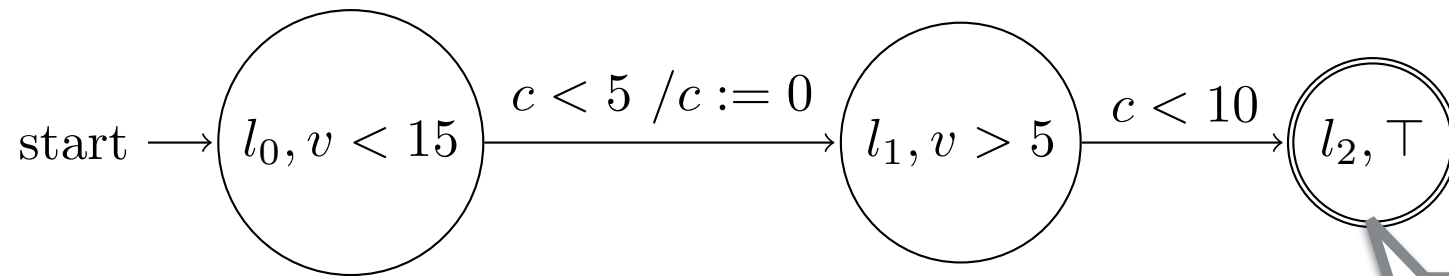
$$\rightarrow (l_0, c=0, 0, \varepsilon) \xrightarrow{2.0} (l_0, c=2, 2, \{v=7\}) \xrightarrow[\kappa(v < 15, \{v=7\})]{e} (l_1, c=0, 2, \varepsilon)$$

One path in Weighted TTS



$$\begin{aligned}
 &\rightarrow (l_0, c=0, 0, \varepsilon) \xrightarrow{2.0} (l_0, c=2, 2, \{v=7\}) \xrightarrow[\kappa(v < 15, \{v=7\})]{e} (l_1, c=0, 2, \varepsilon) \\
 &\quad \xrightarrow{5.0} (l_1, c=5, 7, \{v=7\}\{v=12\})
 \end{aligned}$$

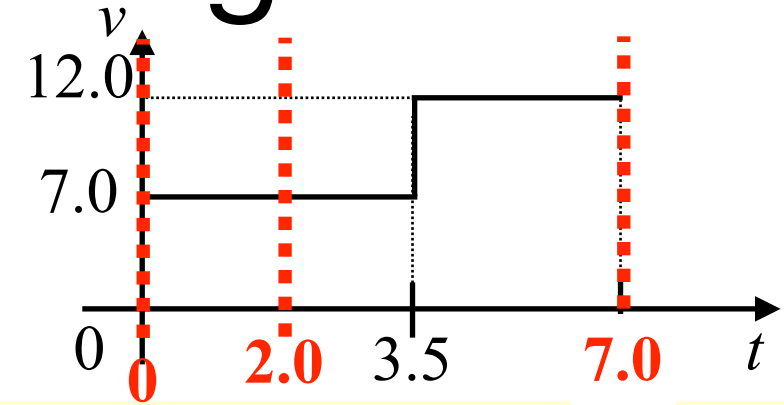
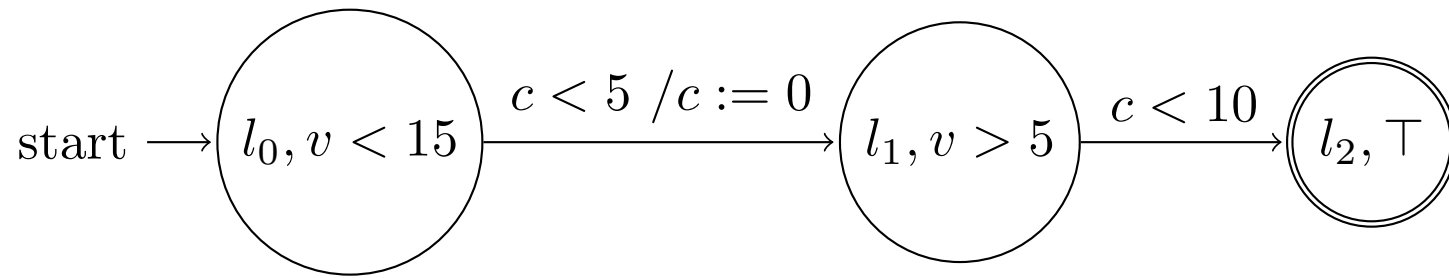
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 &\quad \xrightarrow{5.0} (l_1, c=5, 7, \{v=7\} \{v=12\}) \xrightarrow[\kappa(v > 5, \{v=7\} \{v=12\})]{\otimes e} (l_2, c=5, 7, \varepsilon)
 \end{aligned}$$

	Boolean	sup-inf	tropical
S	{True/False}	$\mathbb{R} \cup \{\pm\infty\}$	$\mathbb{R} \cup \{+\infty\}$
\oplus	\vee	sup	inf
\otimes	\wedge	inf	+

Accumulating paths in Weighted TTS



$$\rightarrow (l_0, c=0, 0, \varepsilon) \xrightarrow{2.0} (l_0, c=2, 2, \{v=7\}) \xrightarrow{e} (l_1, c=0, 3, \varepsilon) \xrightarrow{5.0} (l_1, c=5, 7, \{v=7\}\{v=12\}) \xrightarrow{e} (l_2, c=5, 7, \varepsilon)$$

$$\kappa(v < 15, \{v=7\}) \otimes \kappa(v > 5, \{v=7\}\{v=12\})$$

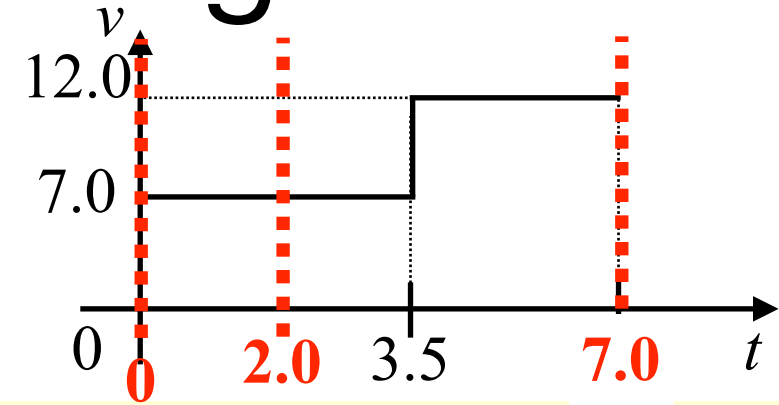
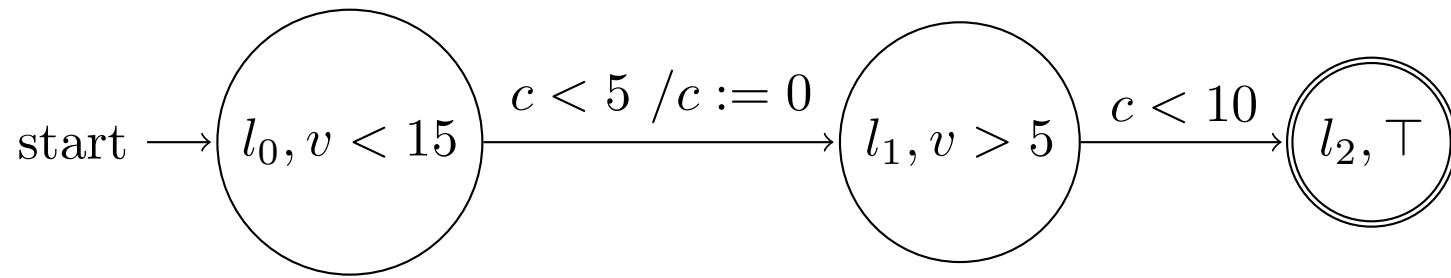
$$\rightarrow (l_0, c=0, 0, \varepsilon) \xrightarrow{4.0} (l_0, c=4, 4, \{v=7\}\{v=12\}) \xrightarrow{e} (l_1, c=0, 4, \varepsilon) \xrightarrow{3.0} (l_1, c=3, 7, \{v=12\}) \xrightarrow{e} (l_2, c=3, 7, \varepsilon)$$

$$\kappa(v < 15, \{v=7\}\{v=12\}) \otimes \kappa(v > 5, \{v=12\})$$

⋮

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\mathcal{S}	{True/False}	$\mathbb{R} \cup \{\pm\infty\}$	$\mathbb{R} \cup \{+\infty\}$
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Accumulating paths in Weighted TTS



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⊕

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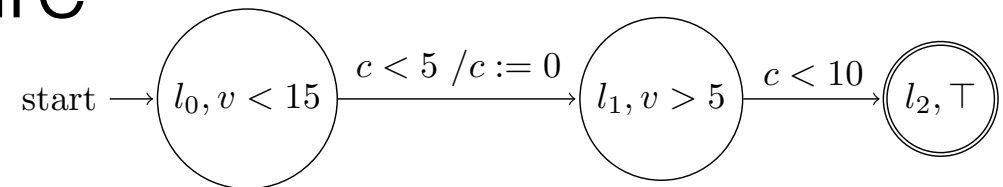
⊕

⋮

	Boolean	sup-inf	tropical
\mathcal{S}	{True/False}	$\mathbb{R} \cup \{\pm\infty\}$	$\mathbb{R} \cup \{+\infty\}$
⊕	v	sup	inf
⊗	∧	inf	+

Timed symbolic weighted automata (TSWA)

- **TSA**: the automata structure



- **Weight function** (κ): the one-step semantics (weight on each transition)
- **Semiring operations** (\otimes, \oplus): how to accumulate weights
One-step semantics \rightarrow semantics for a path/TSWA

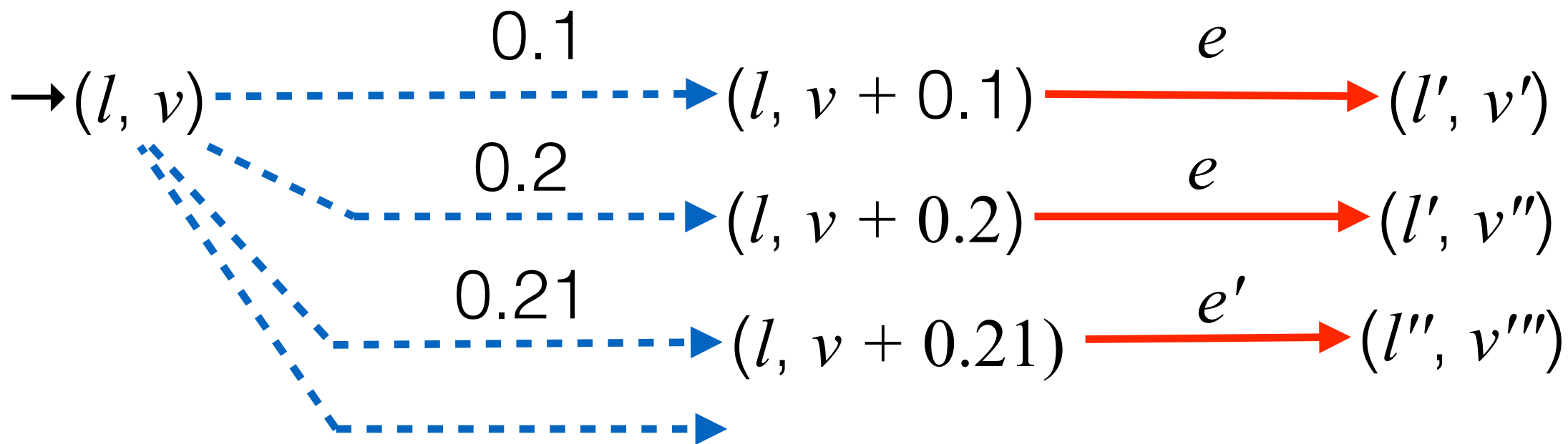
Outline

- Motivation + Introduction
- Technical Part
 - Timed symbolic weighted automata (TSWA)
 - TSWA: TA with signal constraints + weight function
 - Quantitative monitoring/timed pattern matching algorithm
 - Idea: zone construction with weight
- Experiments

Review: Reachability by zones

continuous transitions

discrete transitions

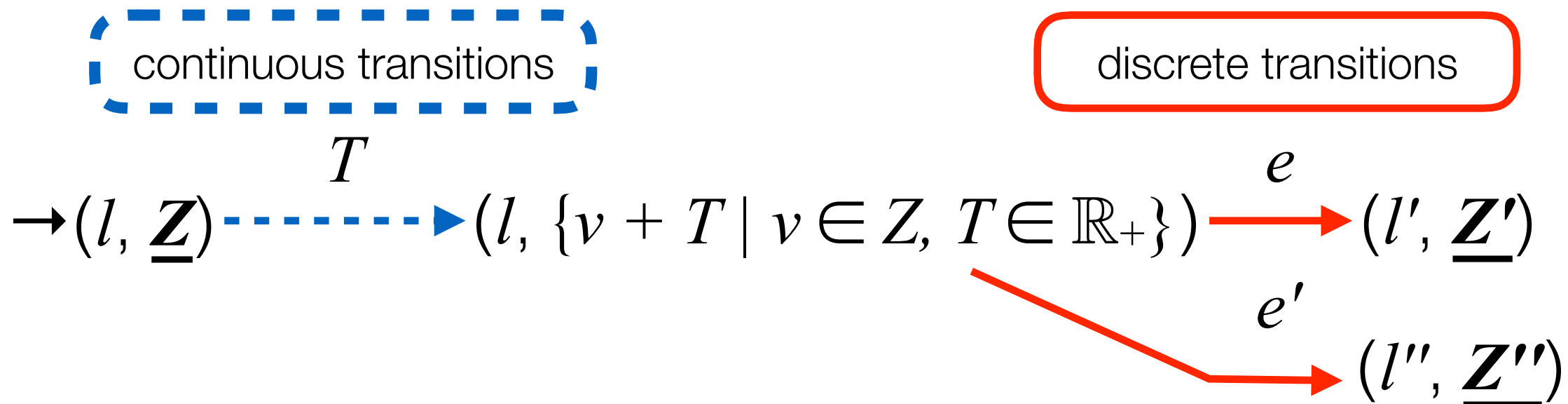


⋮

Infinately many delays!!

Infinately many reachable states!! → symbolic analysis by **zones**

Review: Reachability by zones

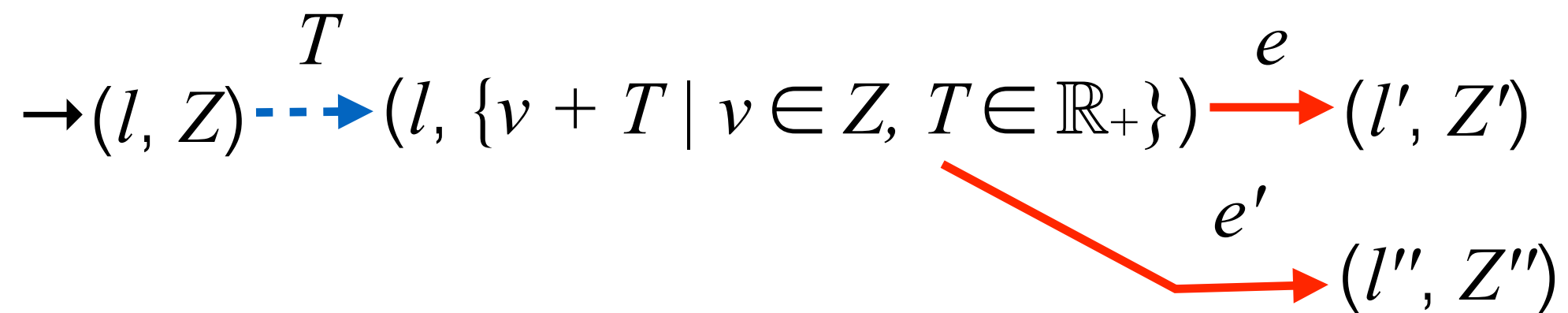


- Zone Z symbolically represents **infinitely many** clock valuations!!

Infinitely many reachable states!! \rightarrow symbolic analysis by **zones**

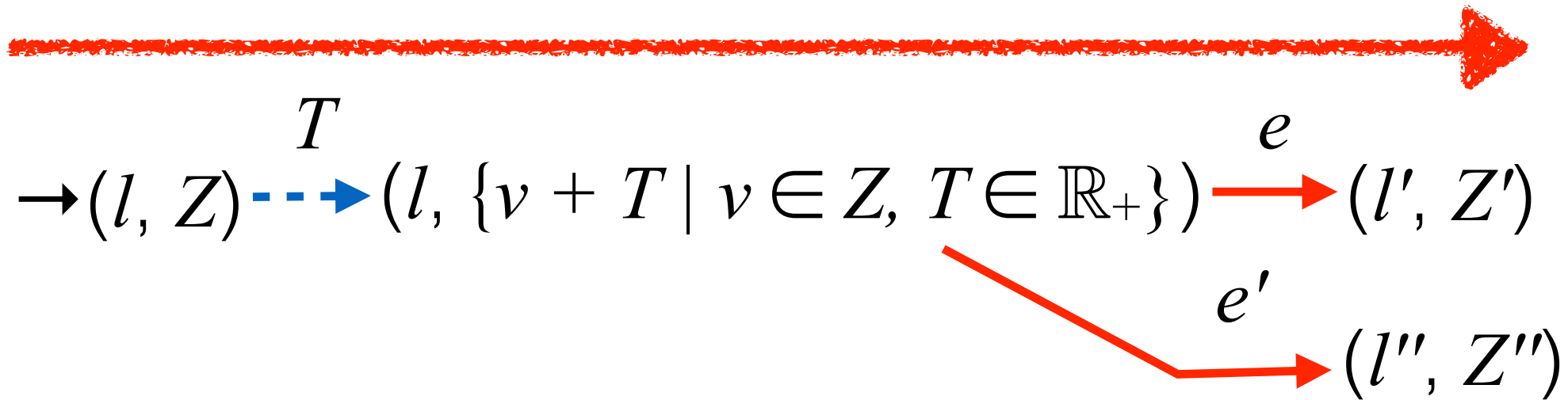
**Observation: reachability
is shortest distance over
Boolean semiring!**

Observation: reachability is shortest distance over Boolean semiring!



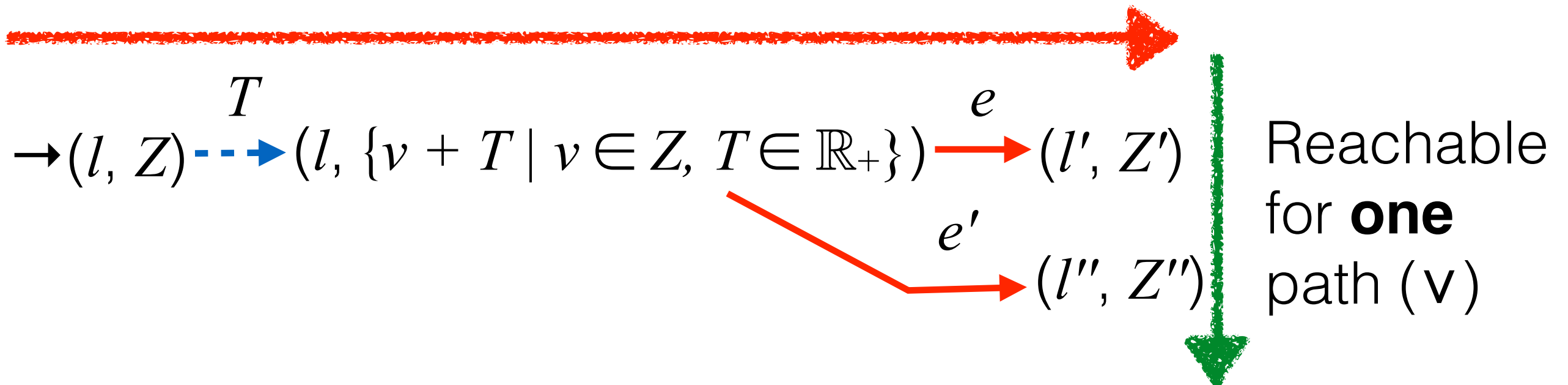
Observation: reachability is shortest distance over Boolean semiring!

Reachable at **all** the transitions (\wedge)



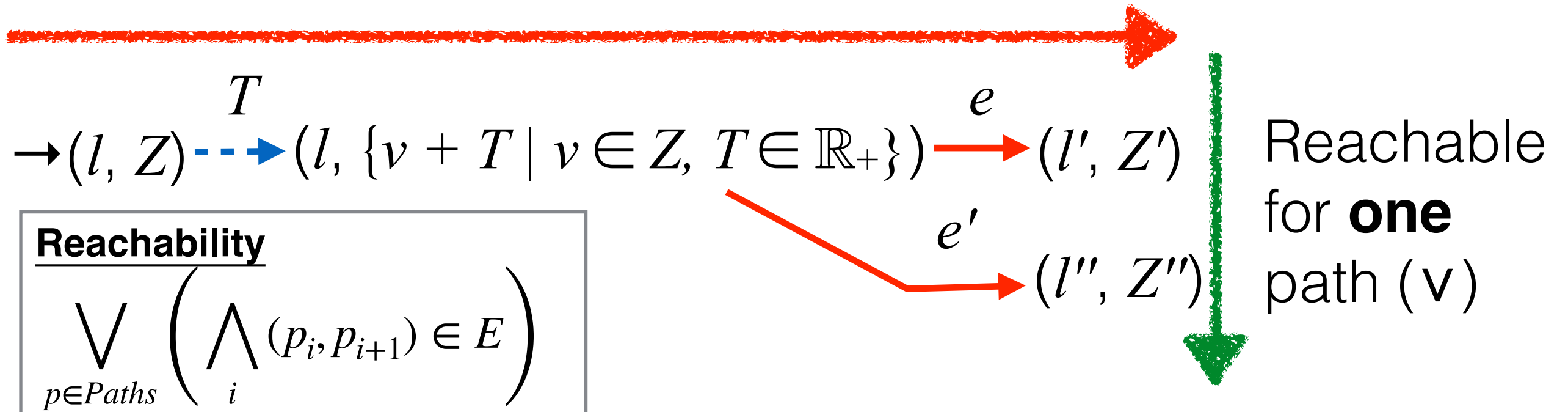
Observation: reachability is shortest distance over Boolean semiring!

Reachable at **all** the transitions (\wedge)



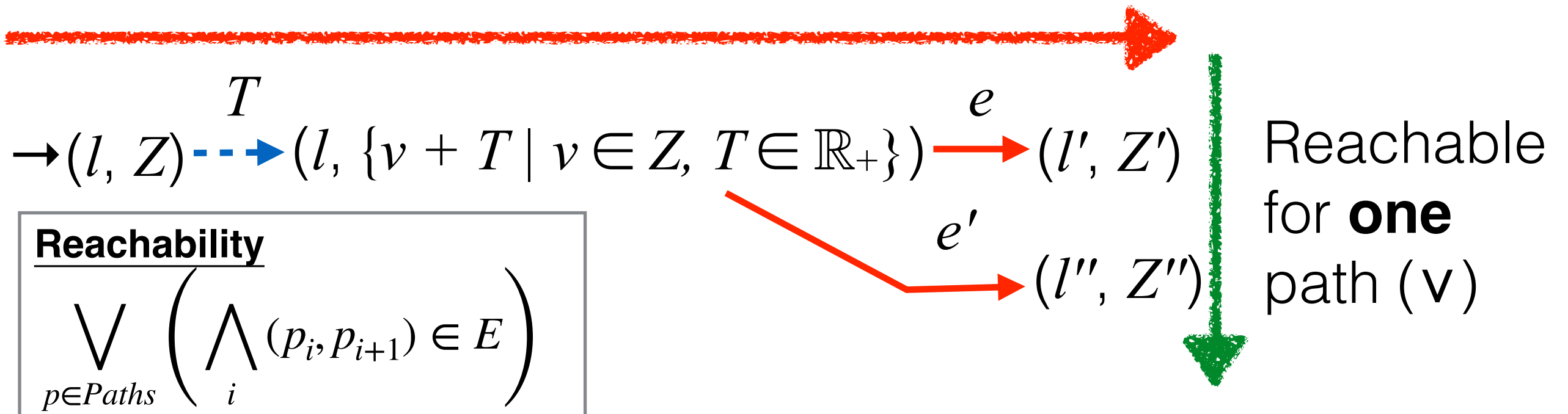
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Reachable at **all** the transitions (\wedge)



Observation: reachability is shortest distance over Boolean semiring!

Reachable at **all** the transitions (\wedge)



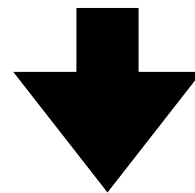
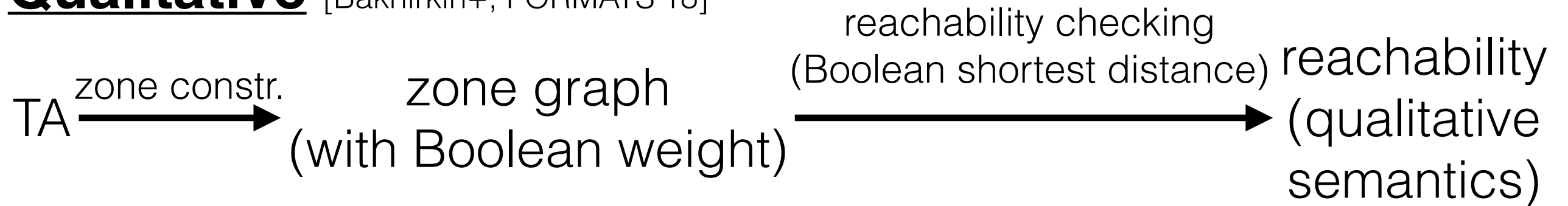
Shortest Distance (for semiring)

$$\bigoplus_{p \in Paths} \left(\bigotimes_i w(p_i, p_{i+1}) \right)$$

	Boolean	sup-inf	tropical
S	{True/False}	$\mathbb{R} \cup \{\pm\infty\}$	$\mathbb{R} \cup \{+\infty\}$
\oplus	\vee	sup	inf
\otimes	\wedge	inf	+

Observation: reachability is shortest distance over Boolean semiring!

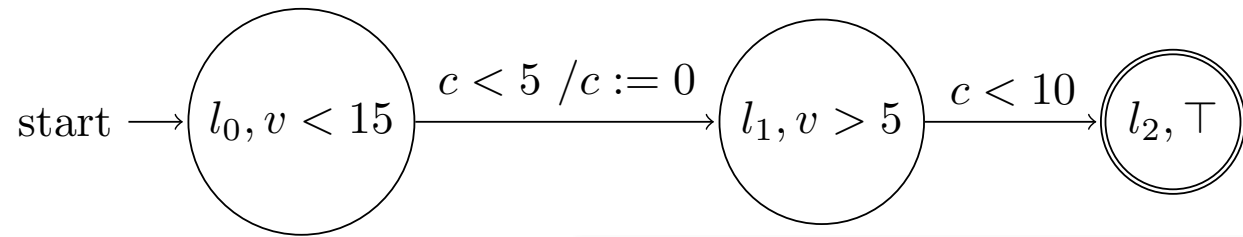
Qualitative [Bakhirkin+, FORMATS'18]



Quantitative [Contribution]

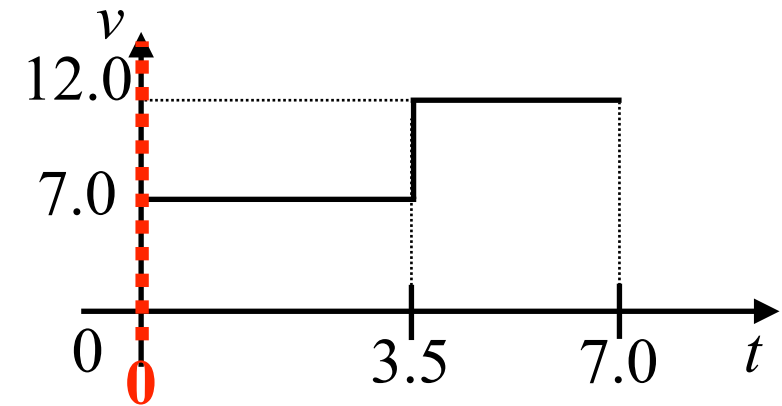


Zone construction with weight



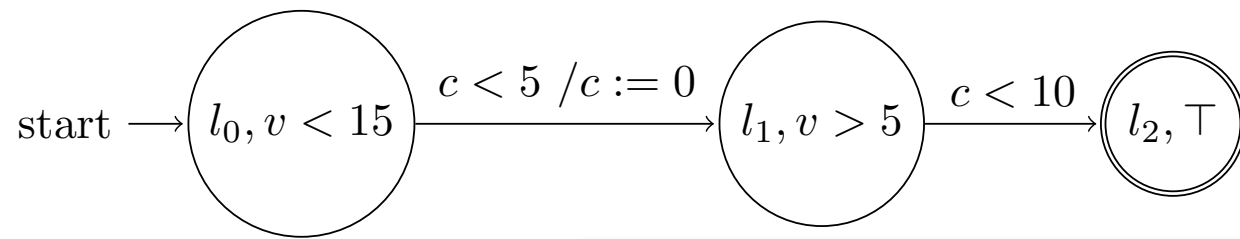
- T : absolute time
- Accepted \Leftrightarrow transit to acc. loc. at $T = |\sigma| (= 7.0)$

This is OK for monitoring



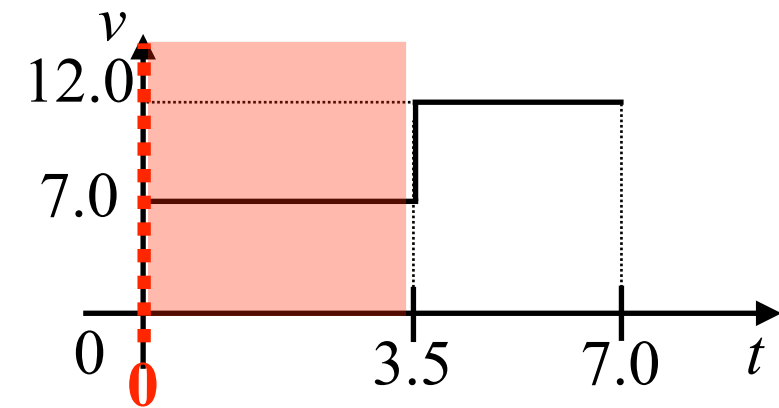
$\rightarrow (l_0, c = T = 0, \varepsilon)$

Zone construction with weight



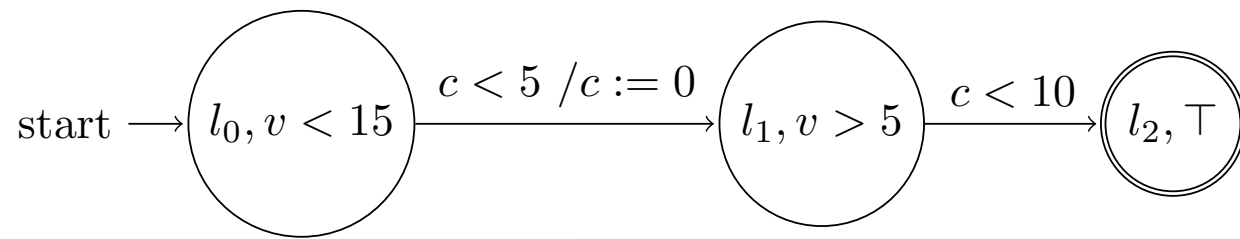
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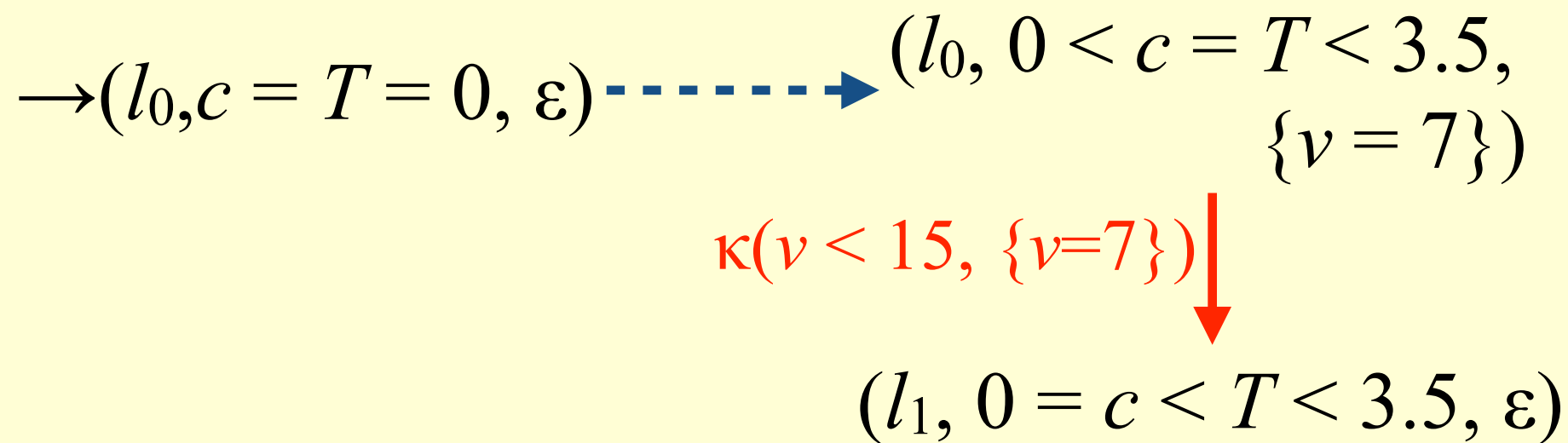
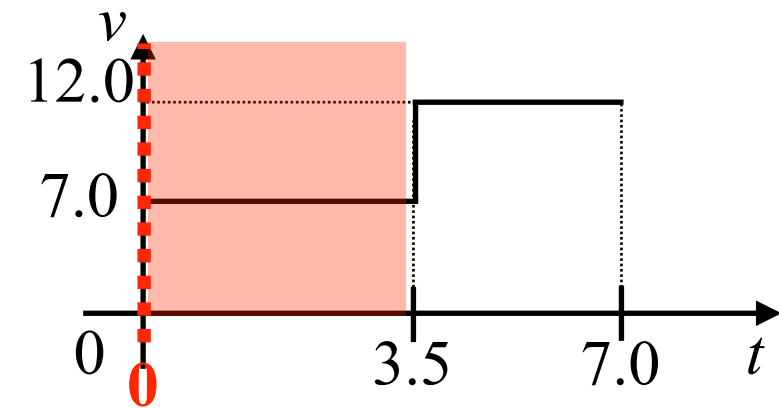
$$\rightarrow (l_0, c = T = 0, \varepsilon) \dashrightarrow (l_0, 0 < c = T < 3.5, \{v = 7\})$$

Zone construction with weight

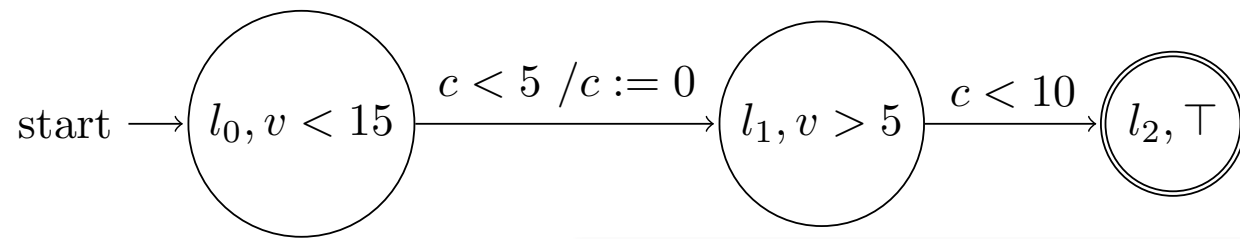


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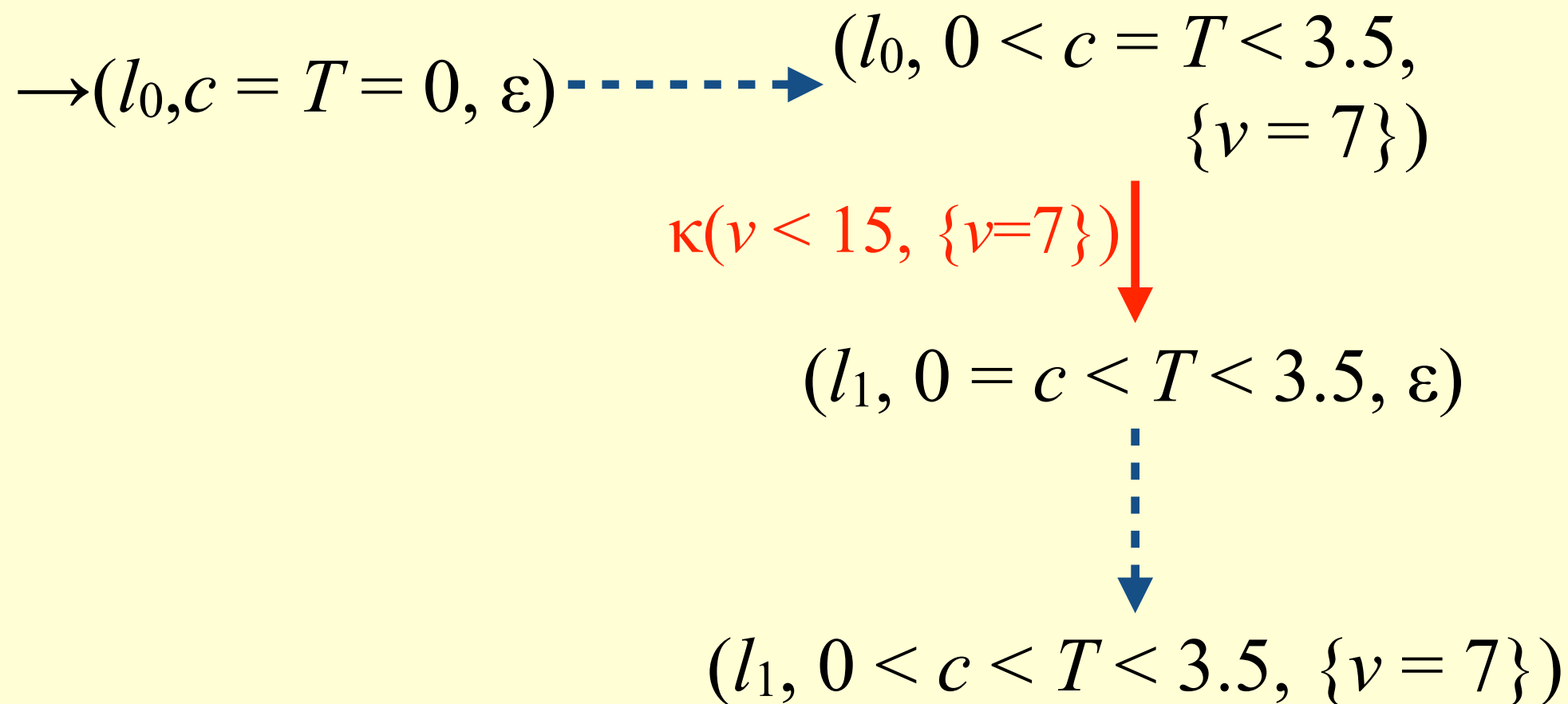
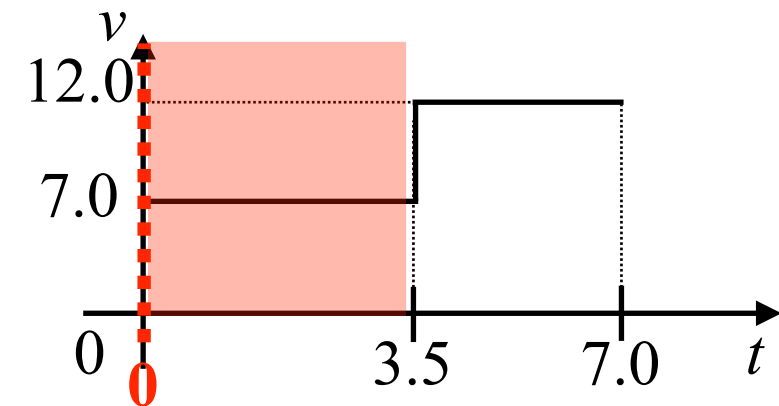


Zone construction with weight

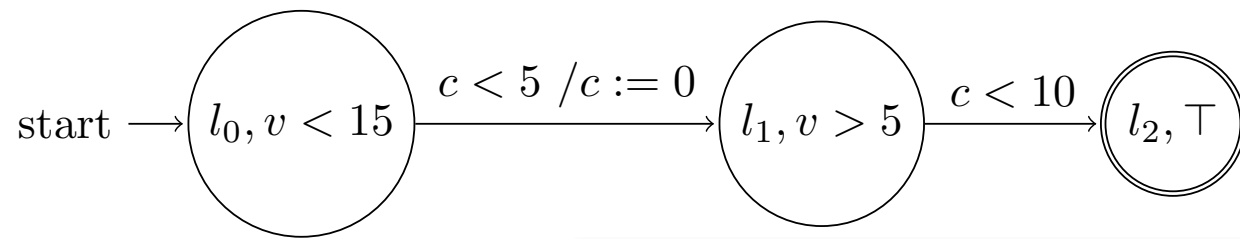


- T : absolute time
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This is OK for monitoring

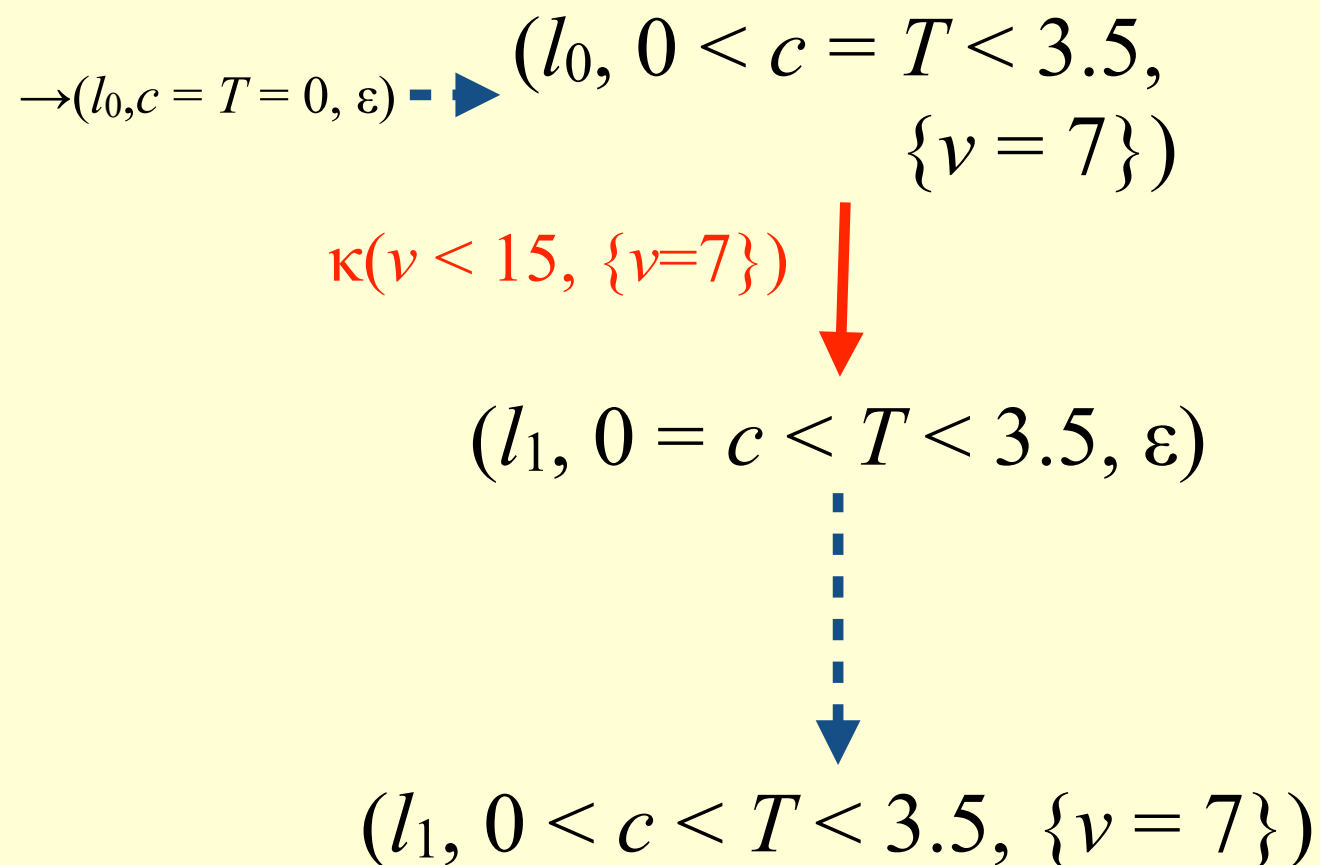
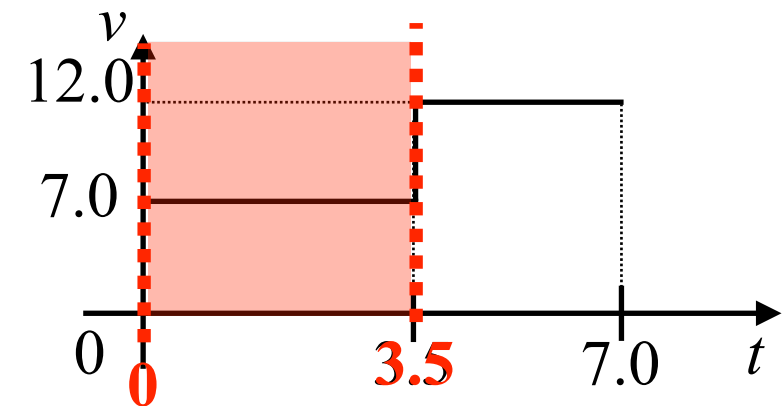


Zone construction with weight

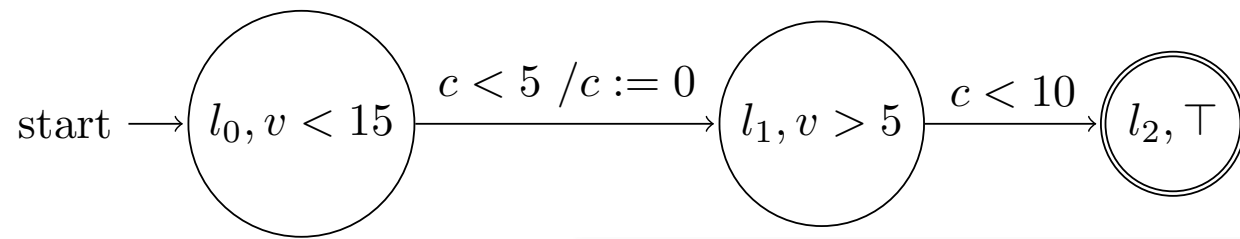


- T : absolute time
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This is OK for monitoring

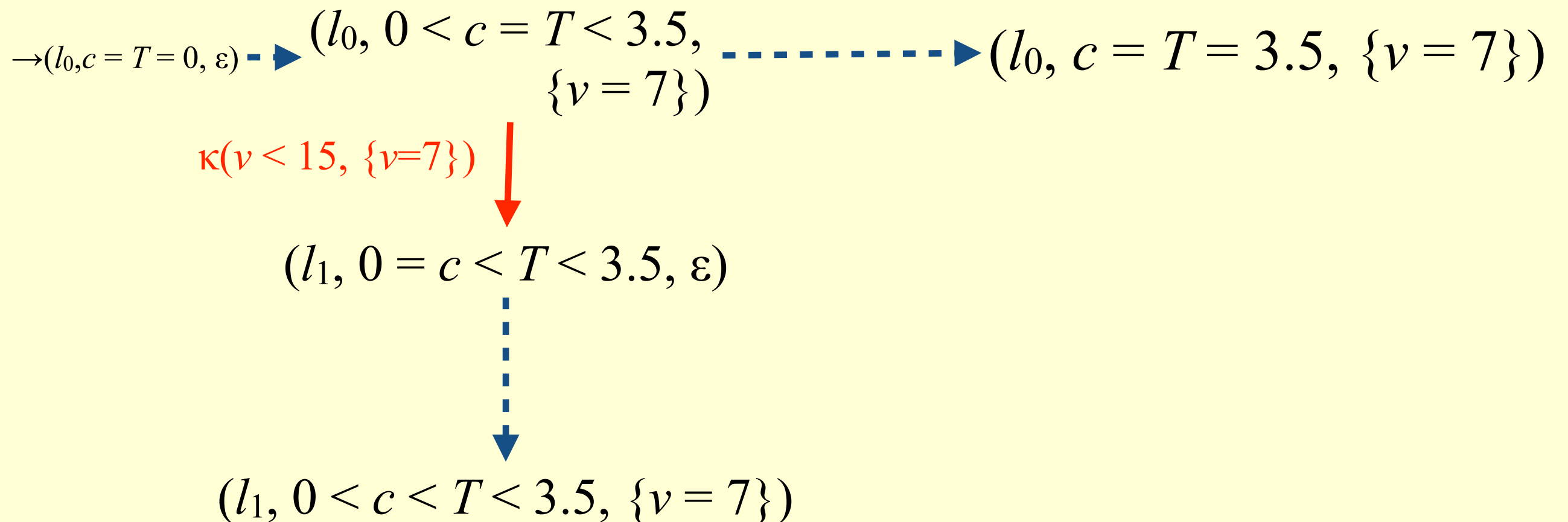
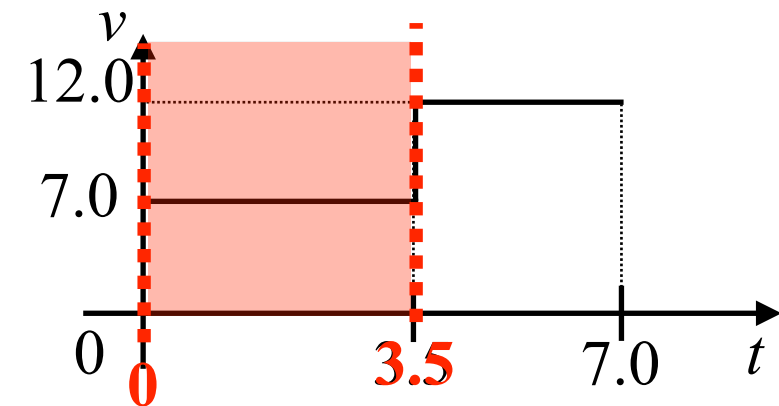


Zone construction with weight

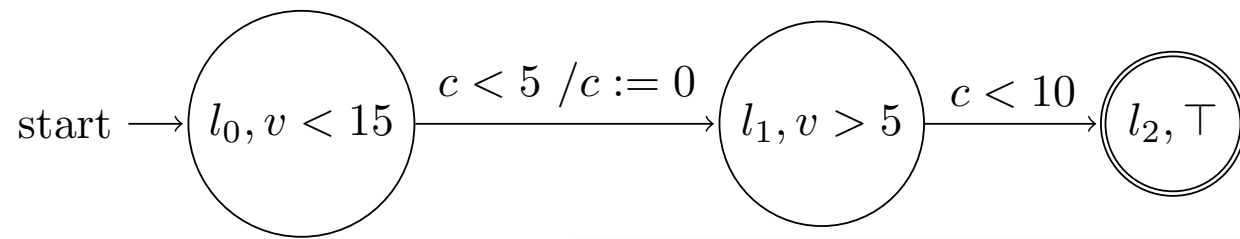


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This is OK for monitoring

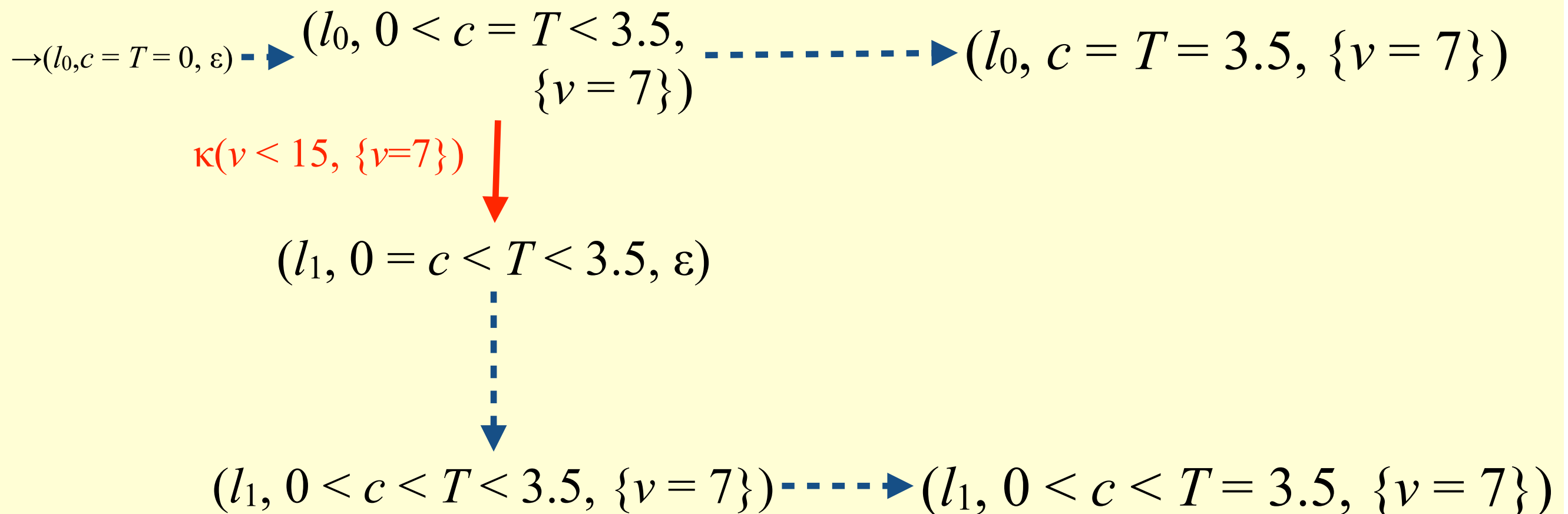
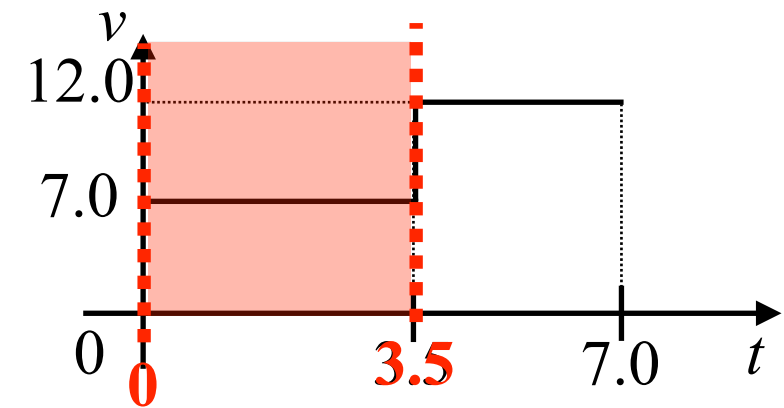


Zone construction with weight

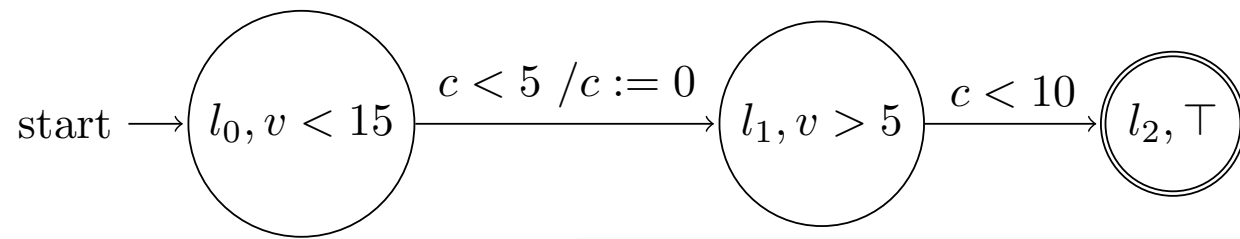


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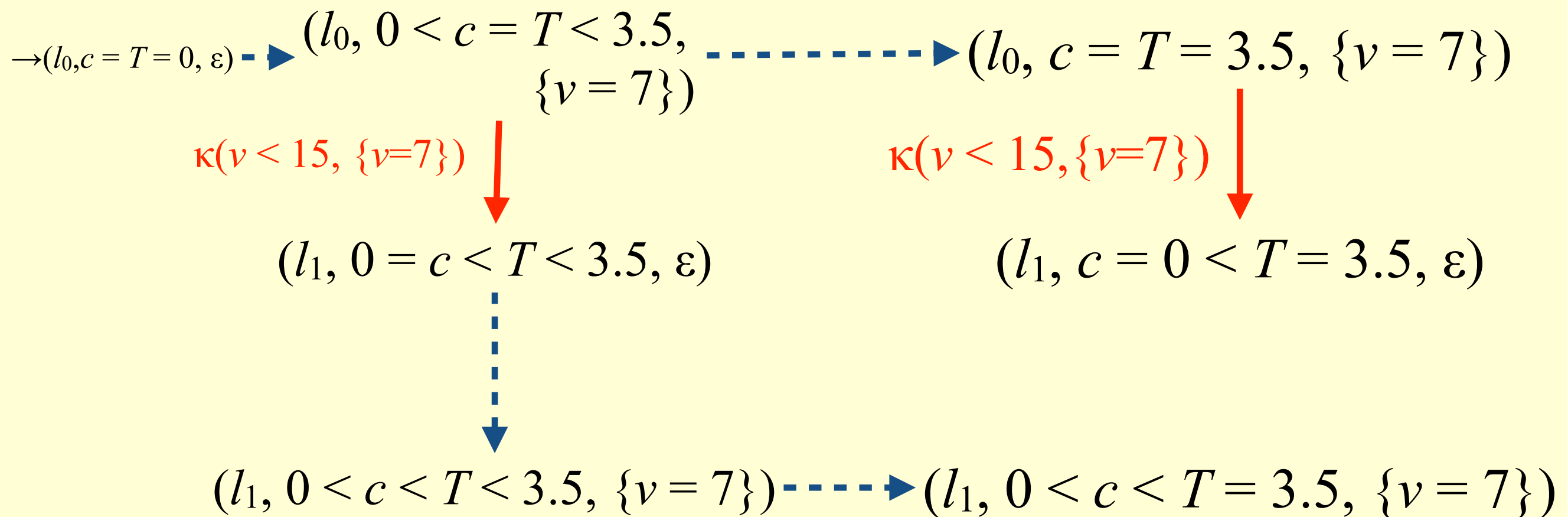
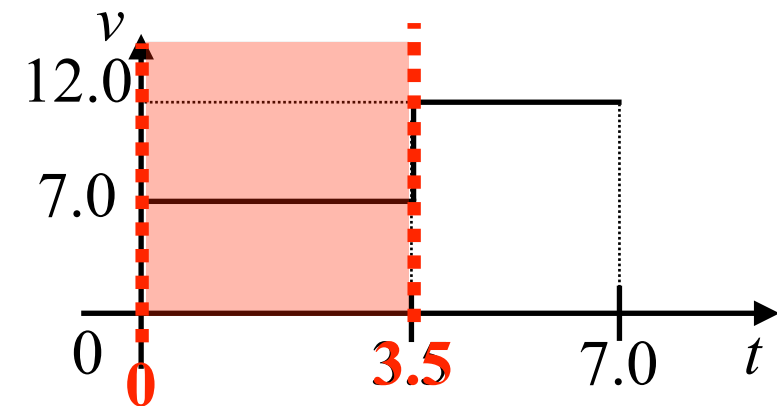


Zone construction with weight

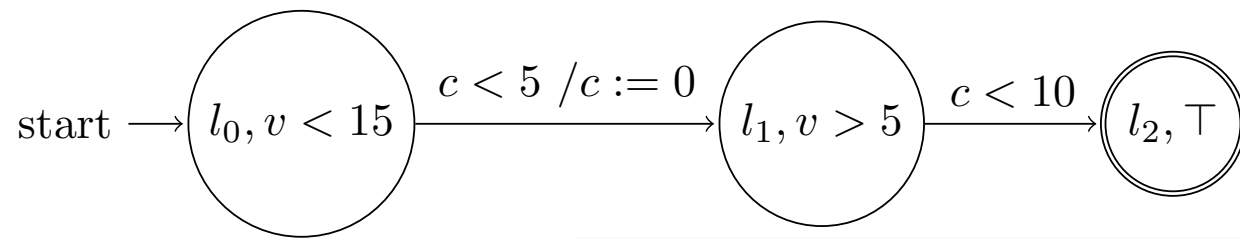


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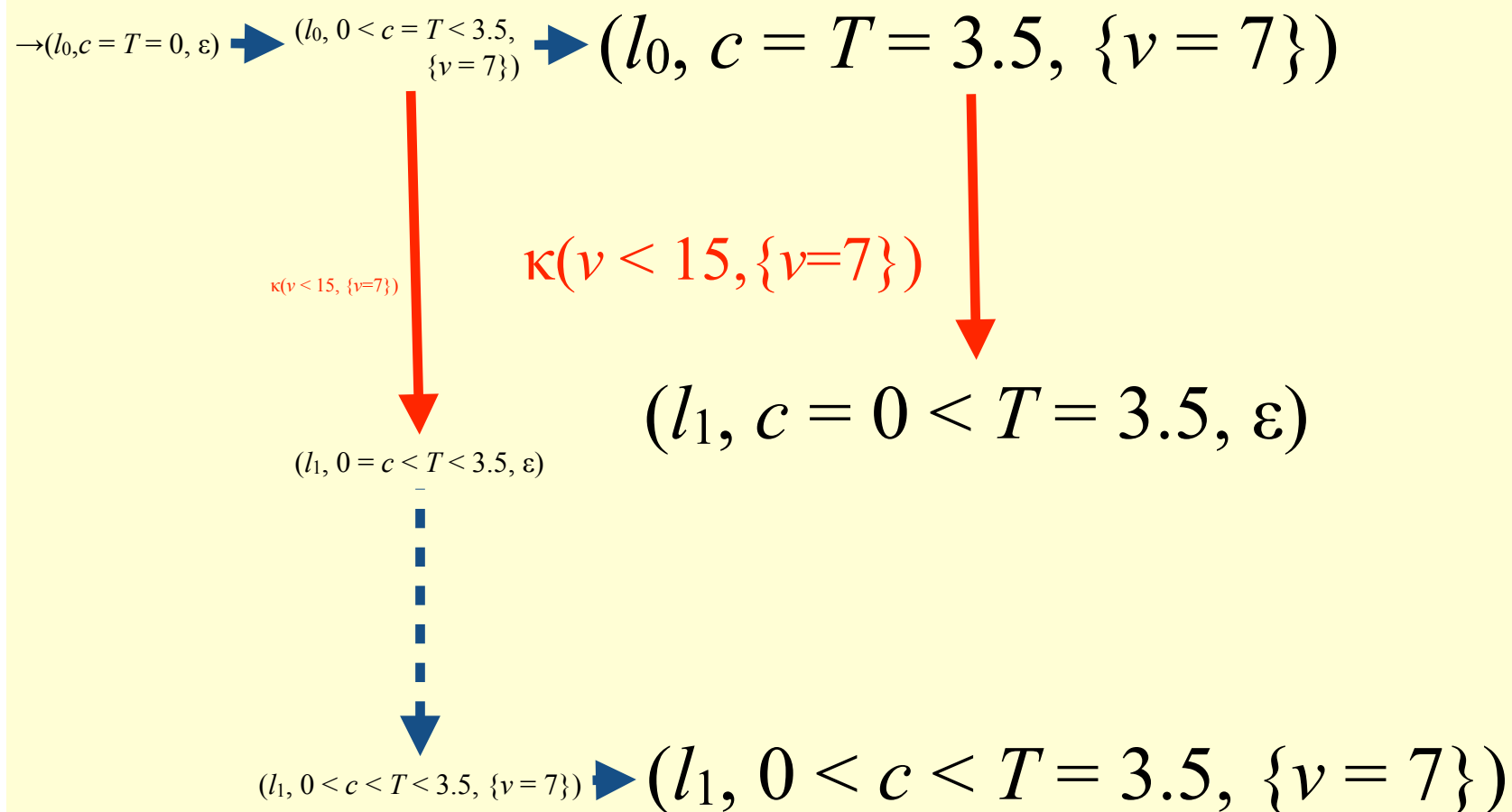
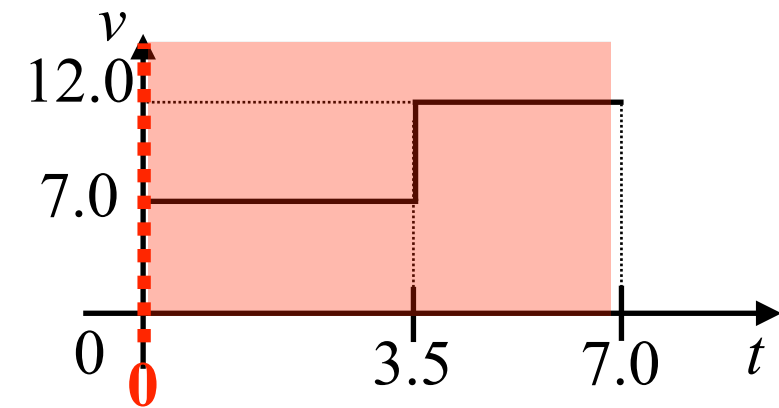


Zone construction with weight

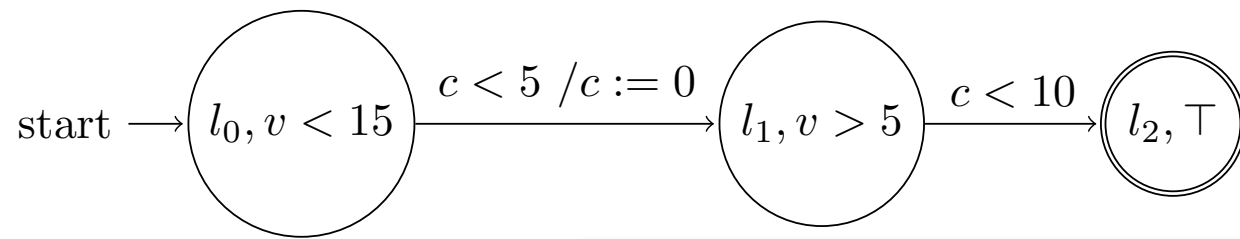


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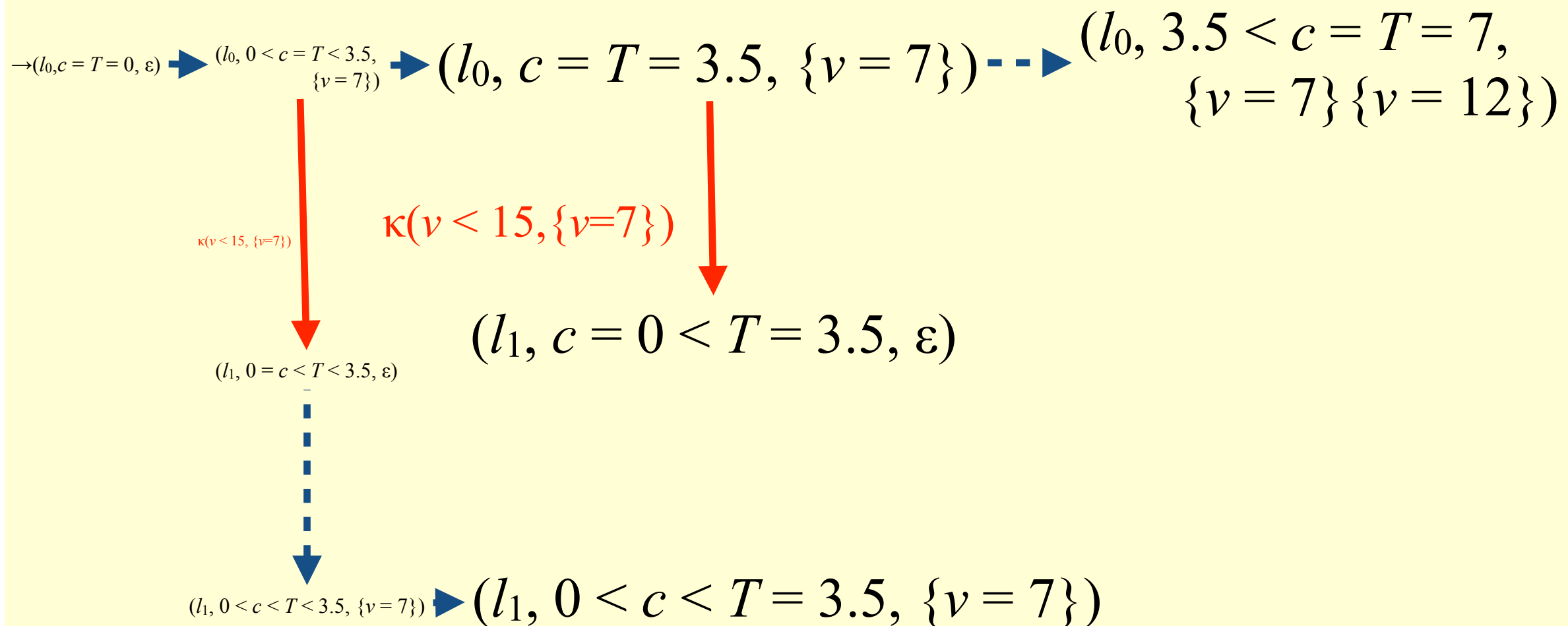
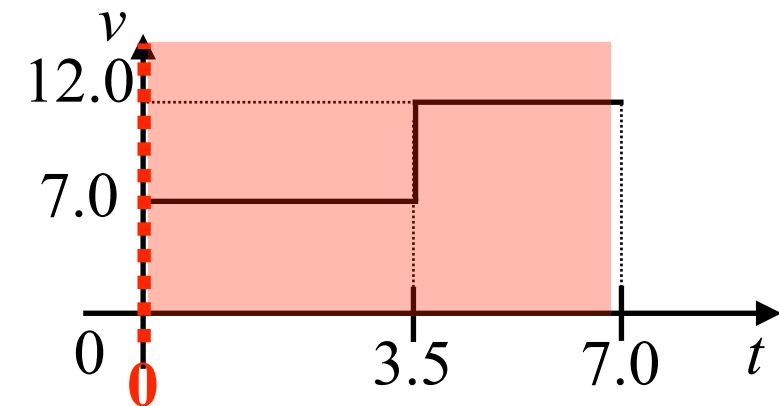


Zone construction with weight

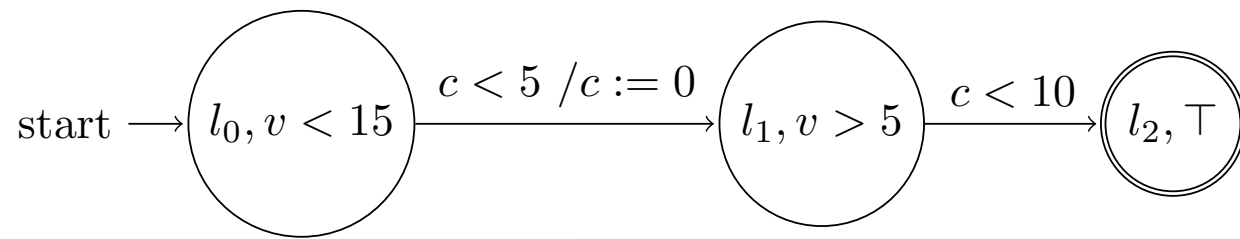


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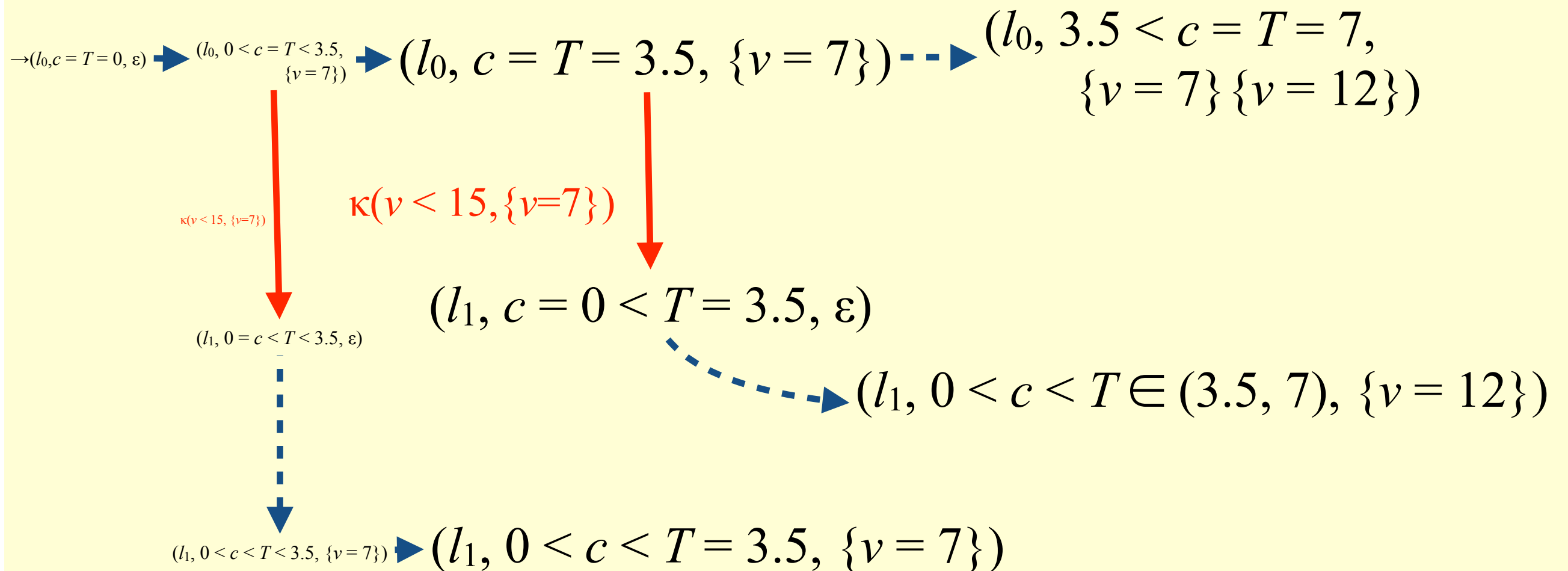
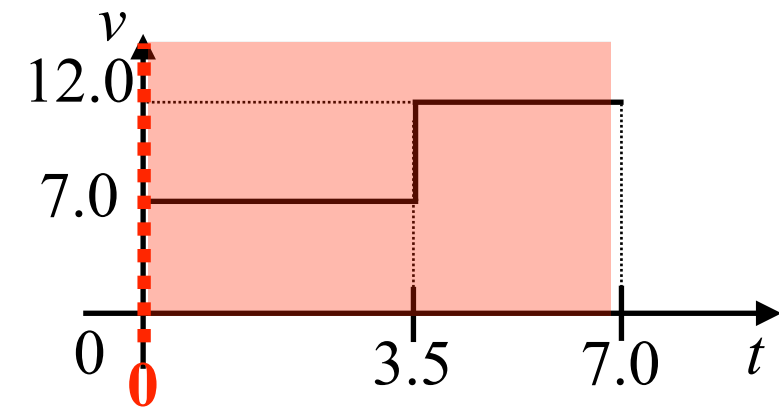


Zone construction with weight

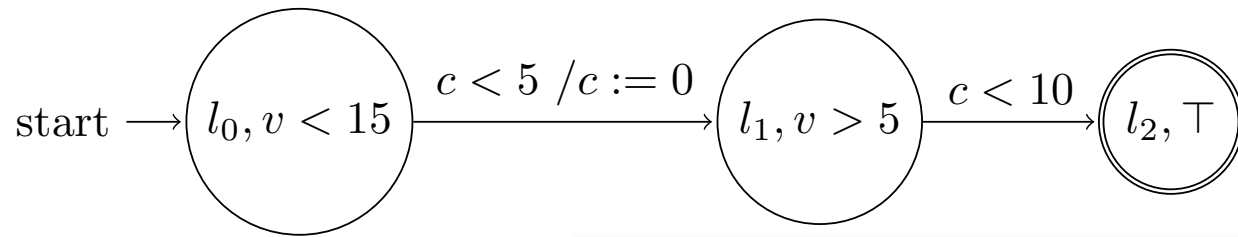


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This is OK for monitoring

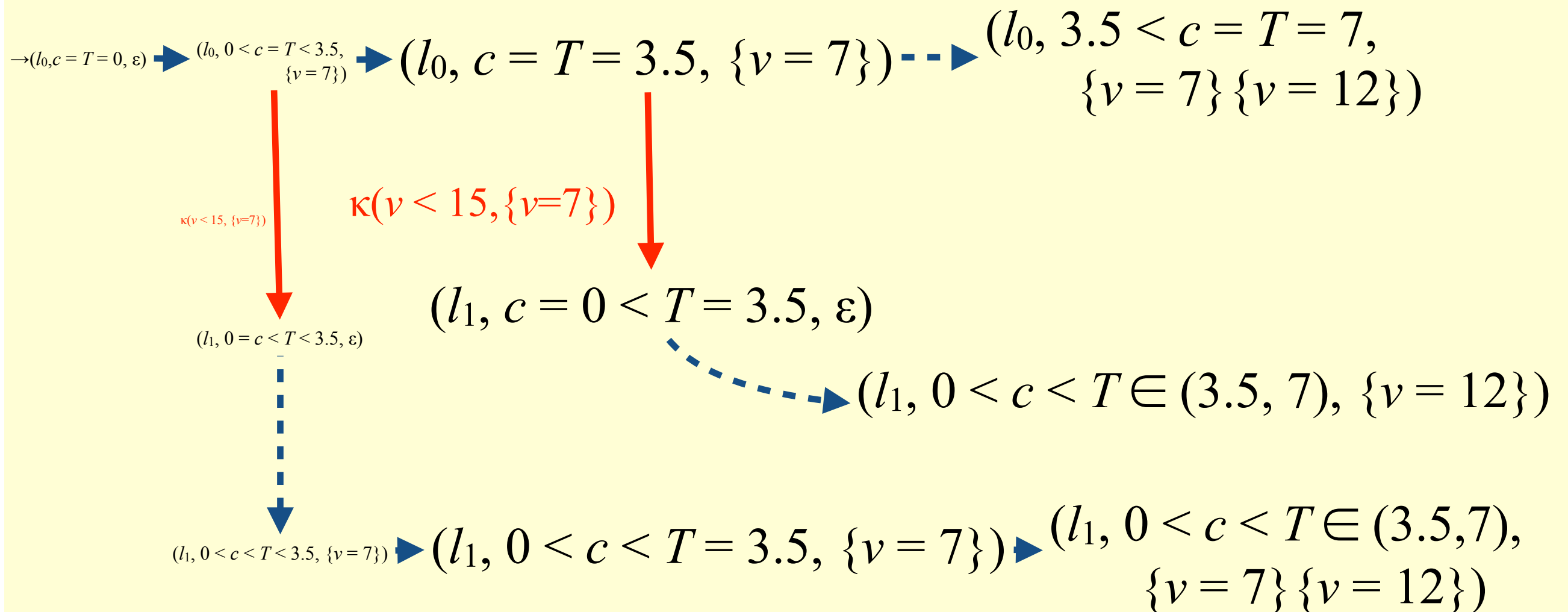
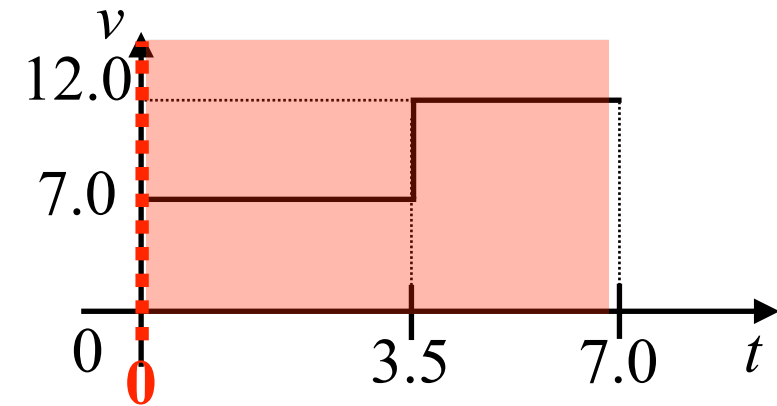


Zone construction with weight

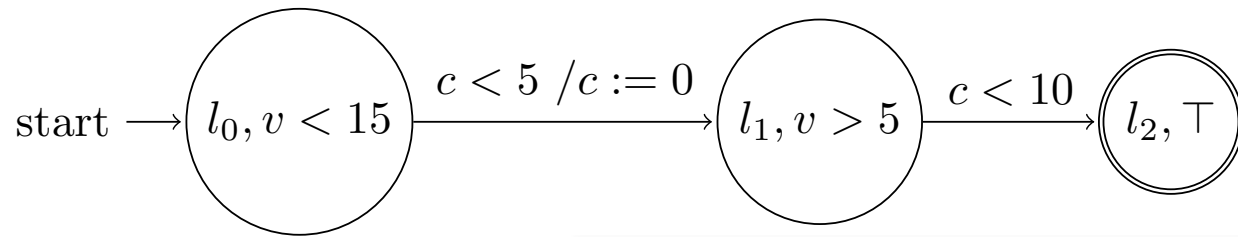


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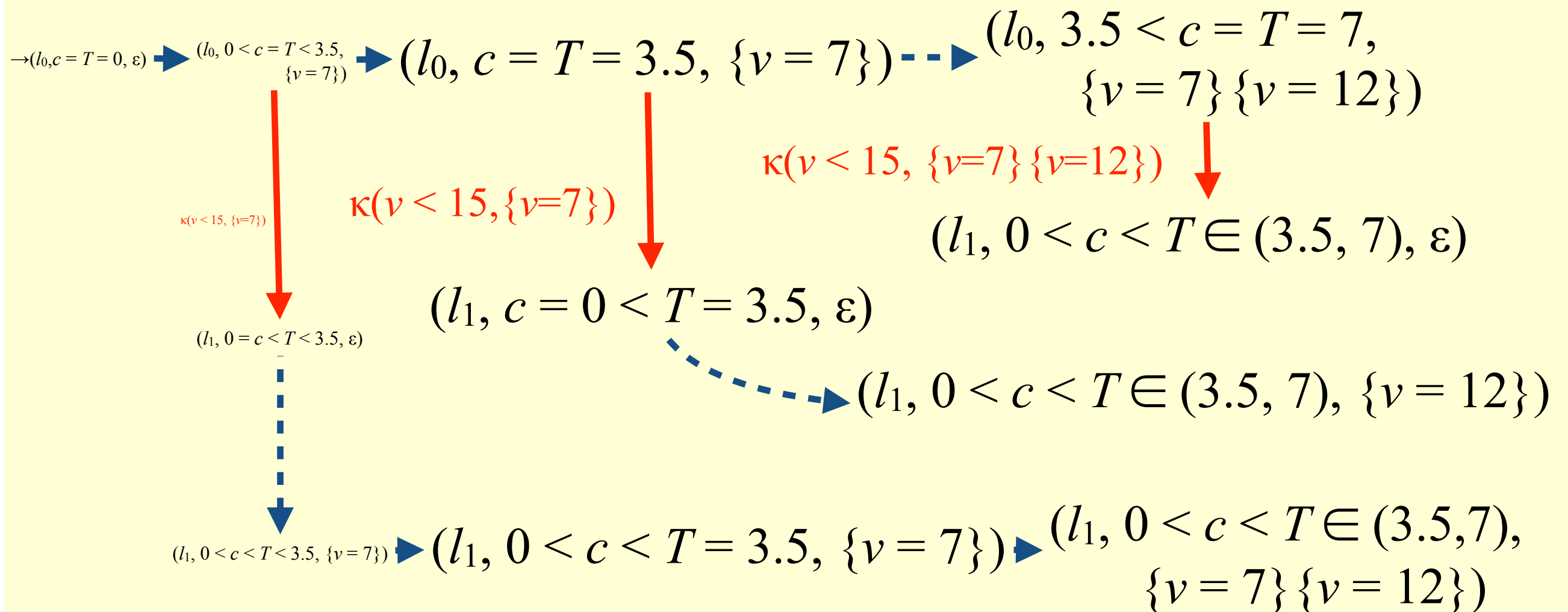
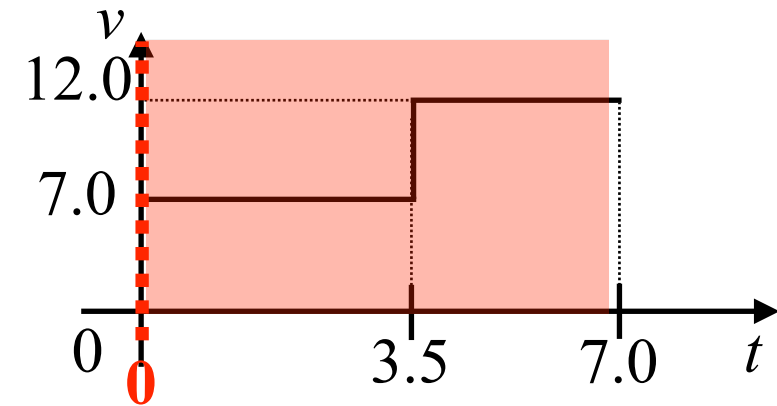


Zone construction with weight

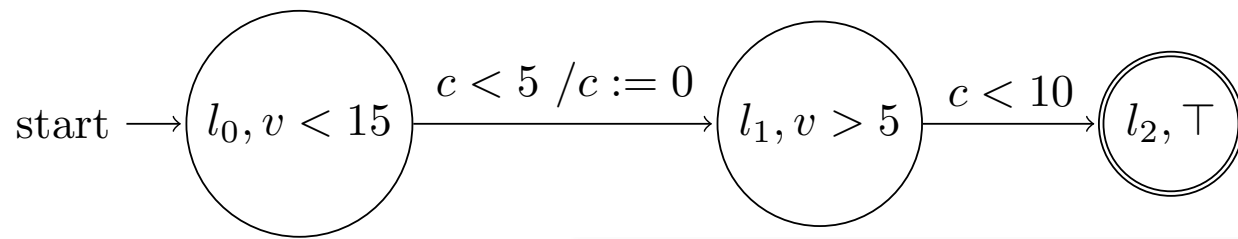


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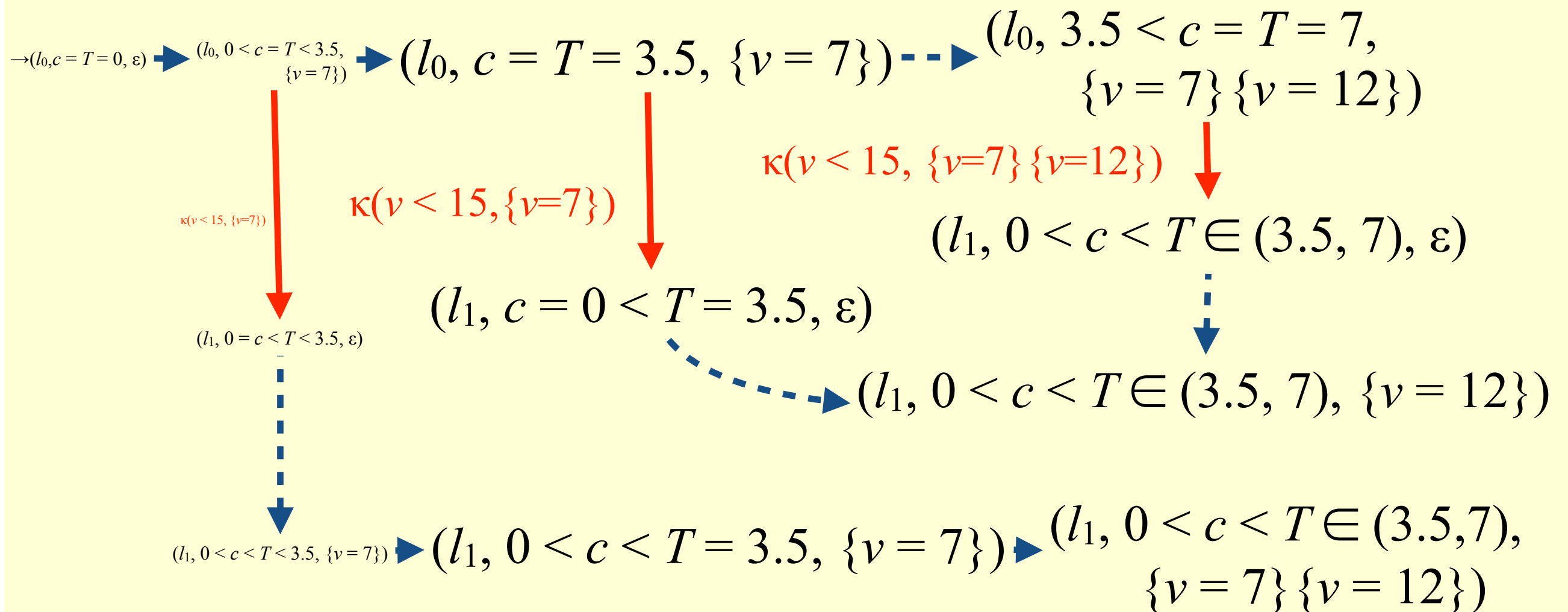
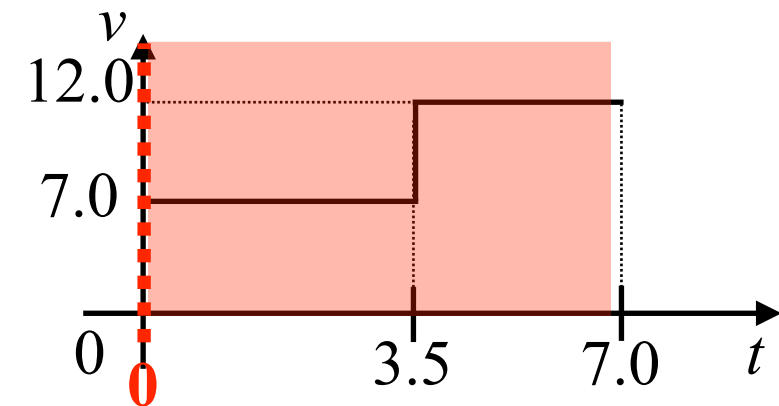


Zone construction with weight

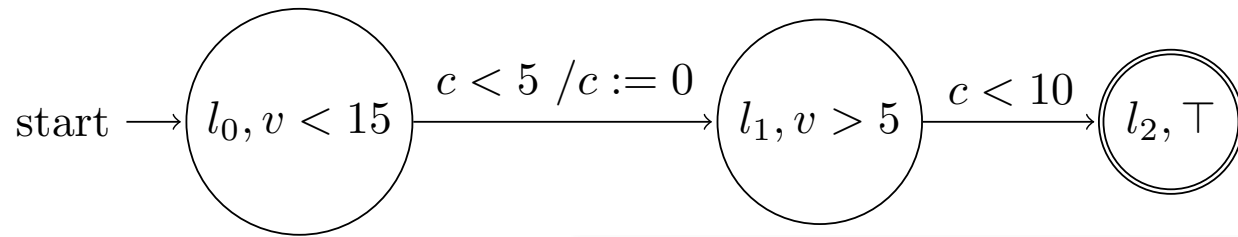


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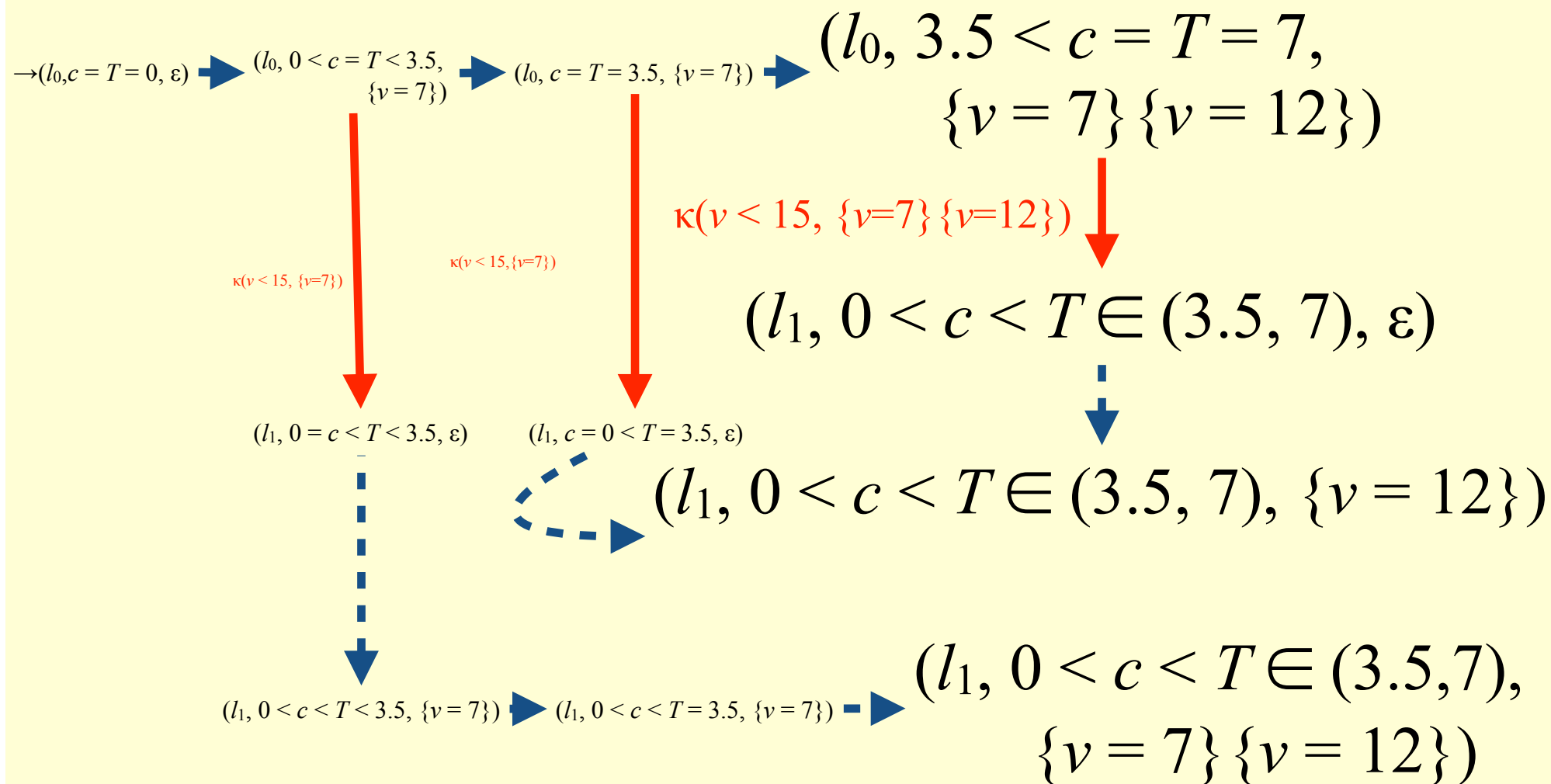
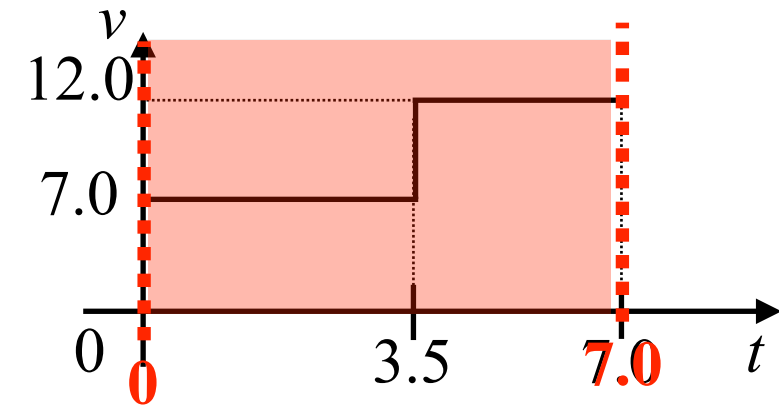


Zone construction with weight

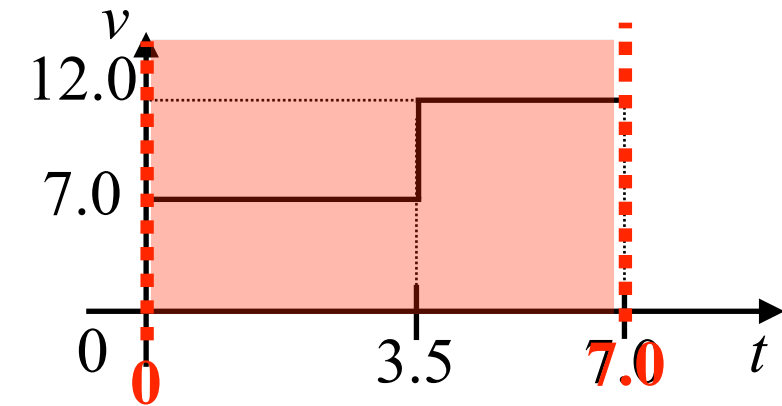
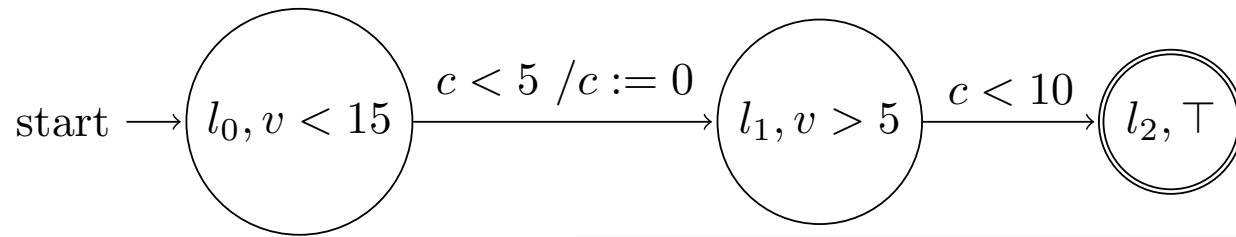


- T : absolute time
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This is OK for monitoring

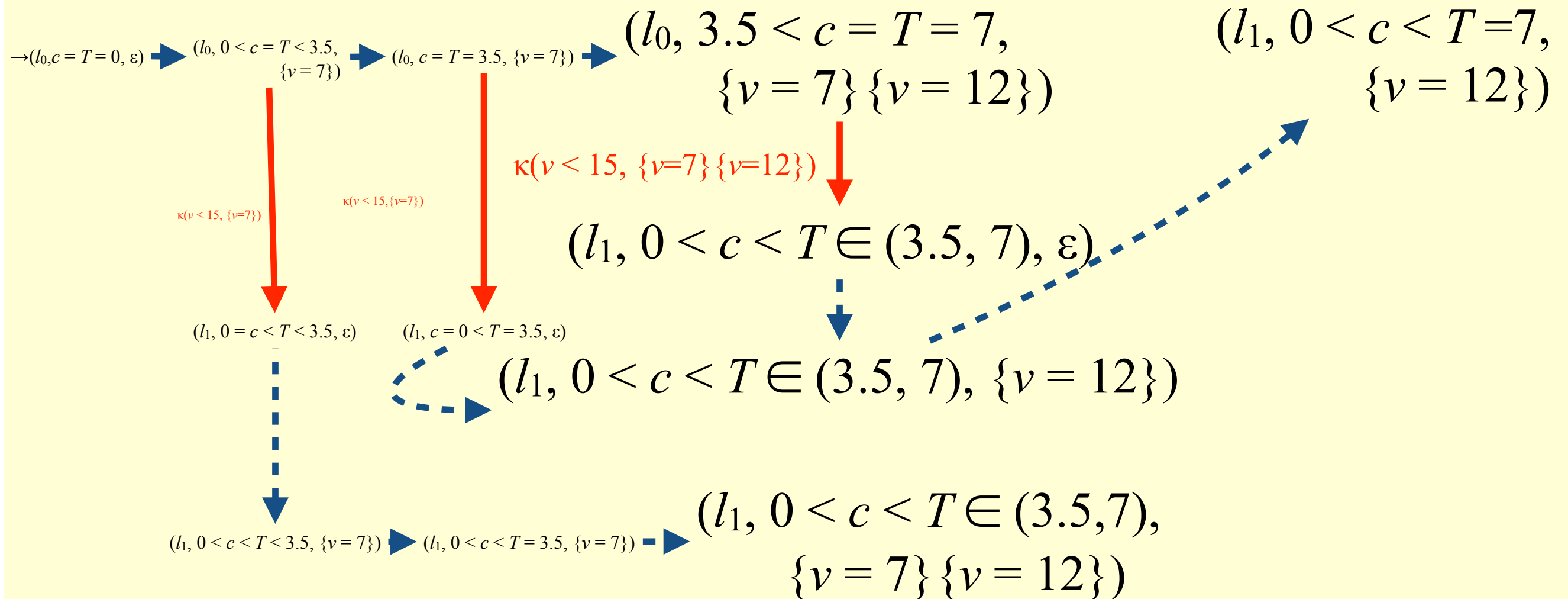


Zone construction with weight

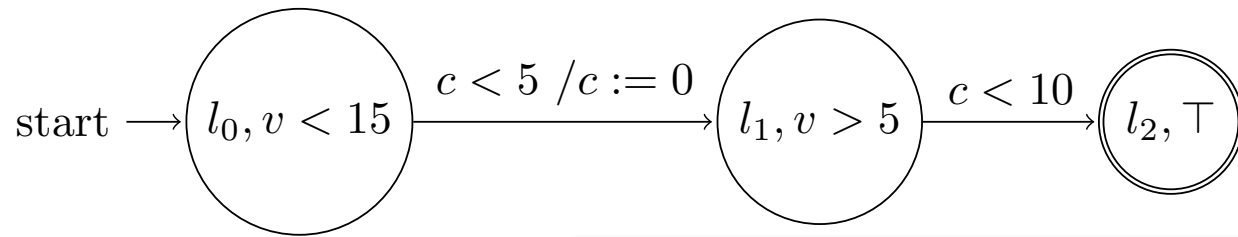


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This is OK for monitoring

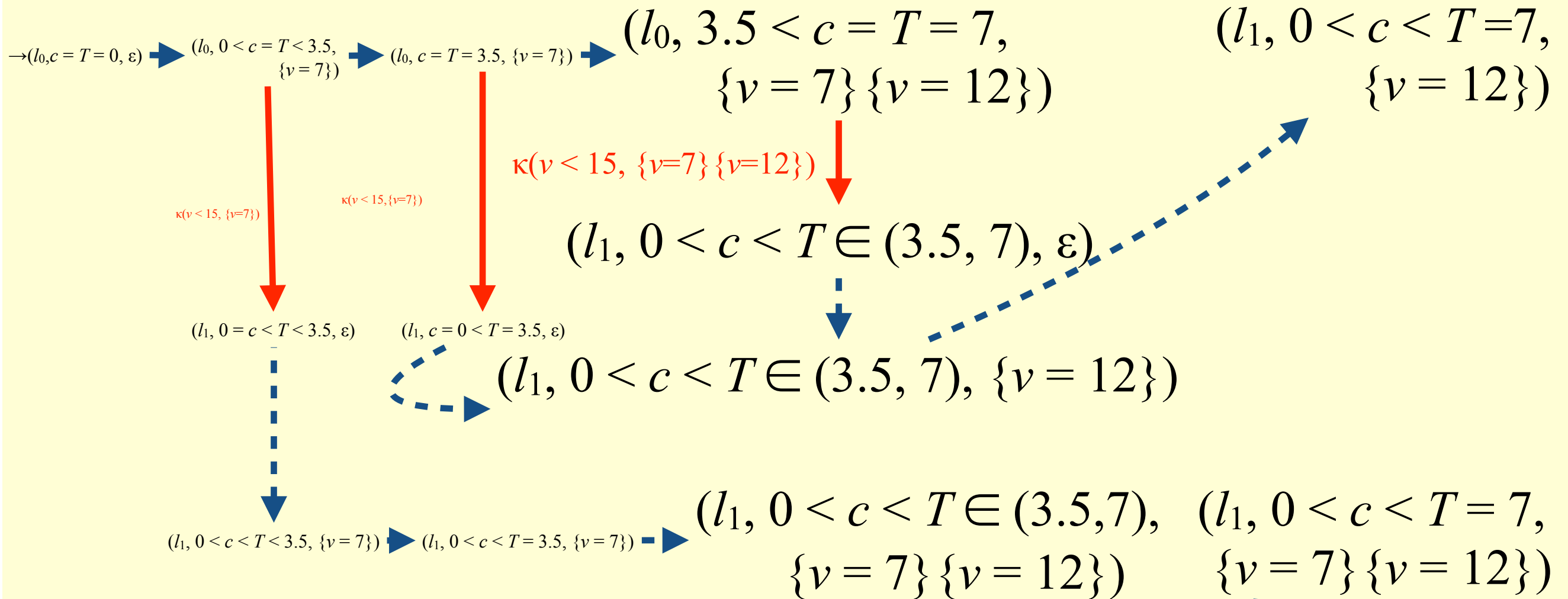
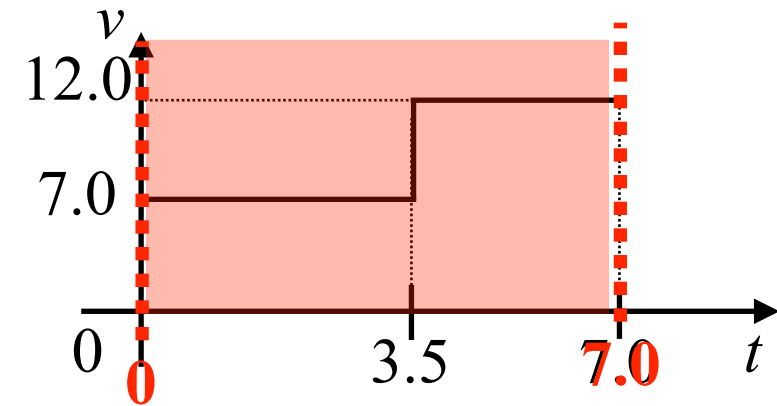


Zone construction with weight

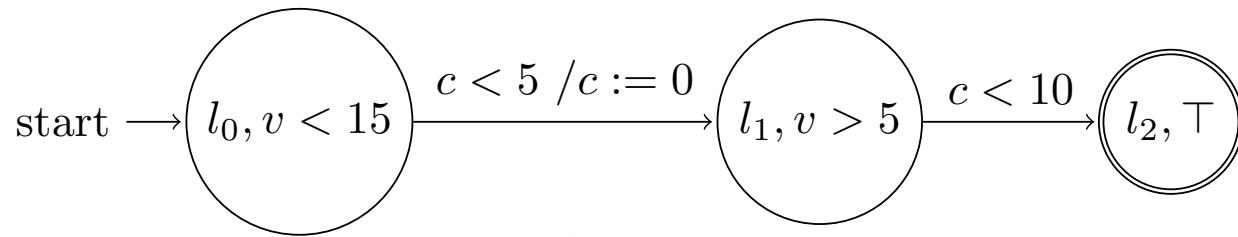


This is OK for monitoring

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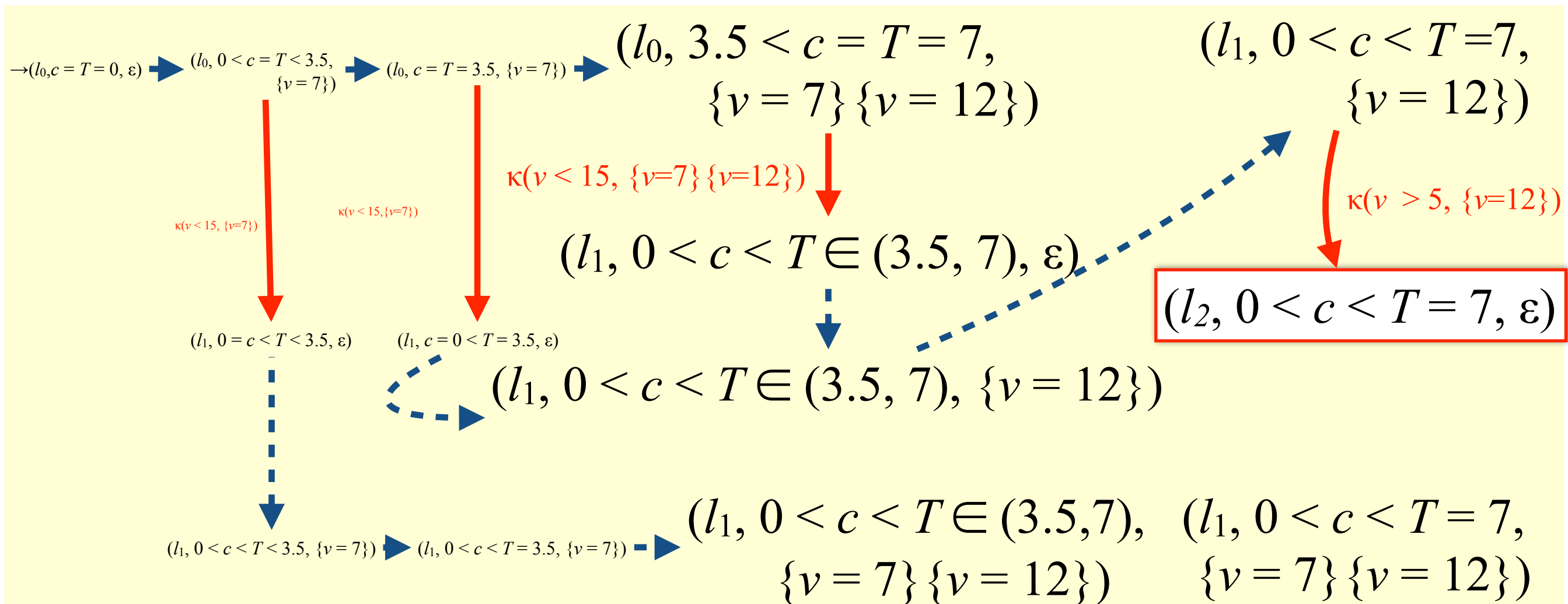
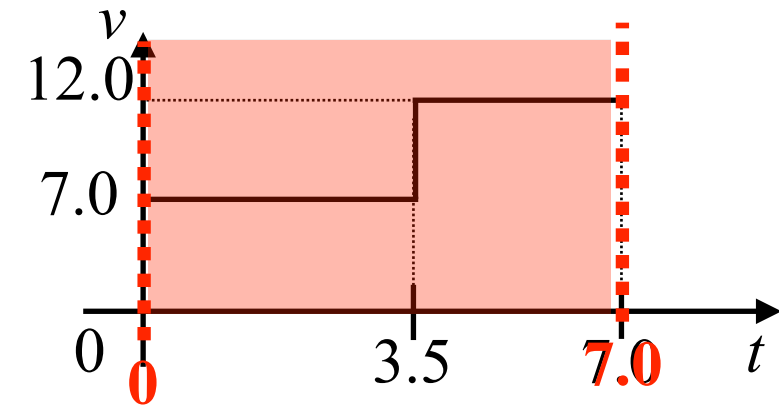


Zone construction with weight

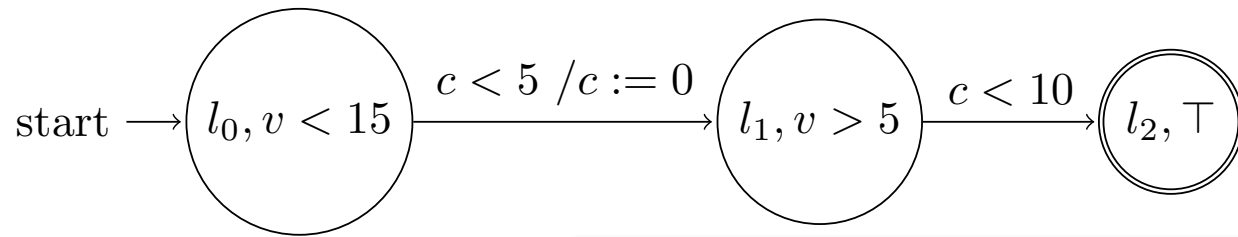


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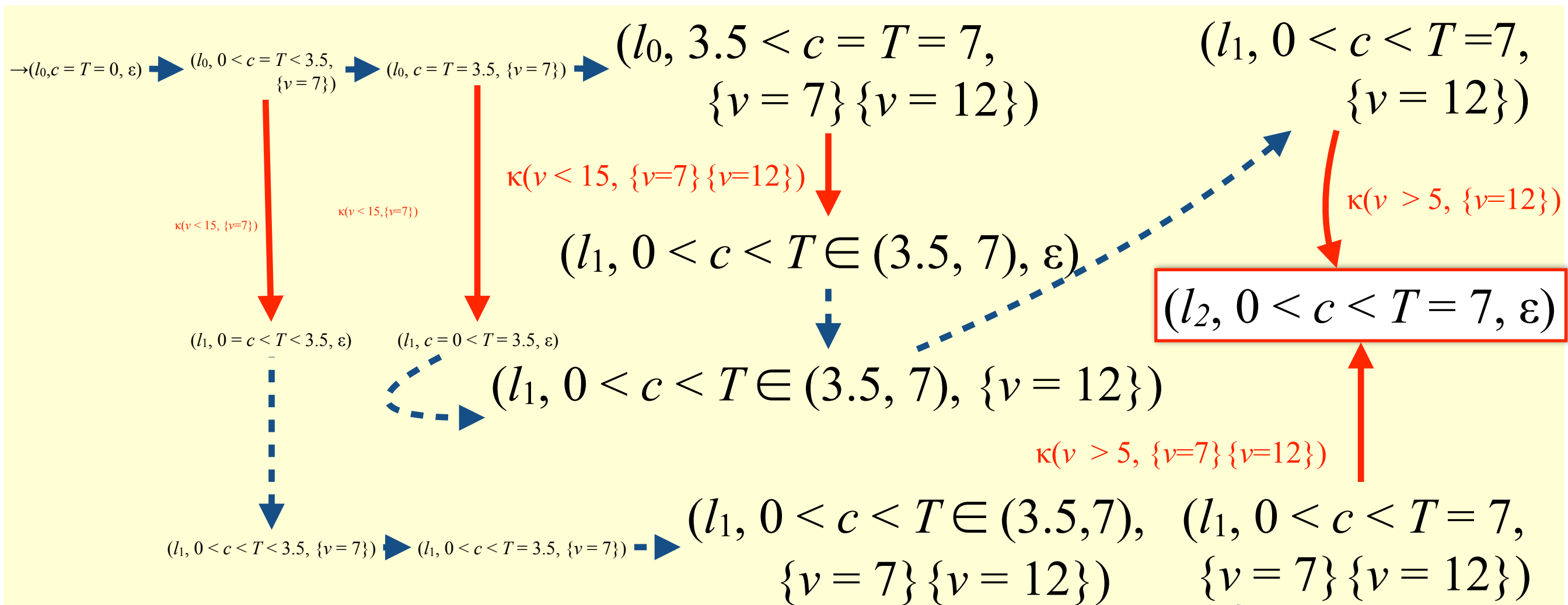
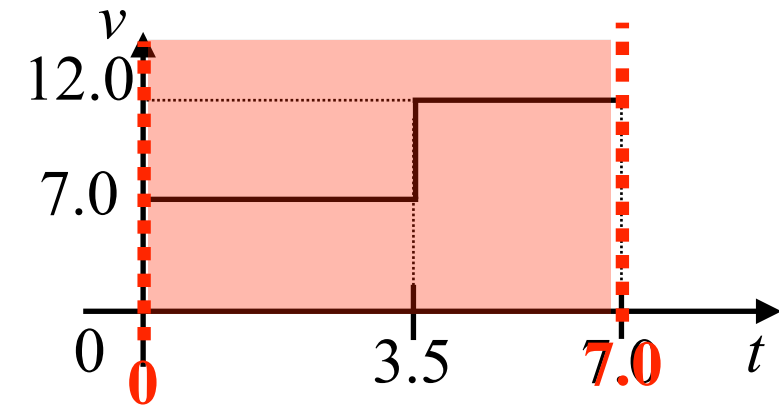


Zone construction with weight



- T : absolute time
- Accepted \Leftrightarrow transit to acc. loc. at $T = |\sigma| (= 7.0)$

This is OK for monitoring



Main Theorem: Correctness

Thm.

The shortest distance in the zone graph with weight is same as the shortest distance in the weighted TTS for any complete and idempotent semiring.

All of them work!!

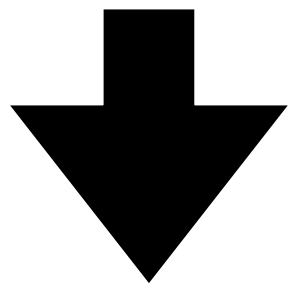
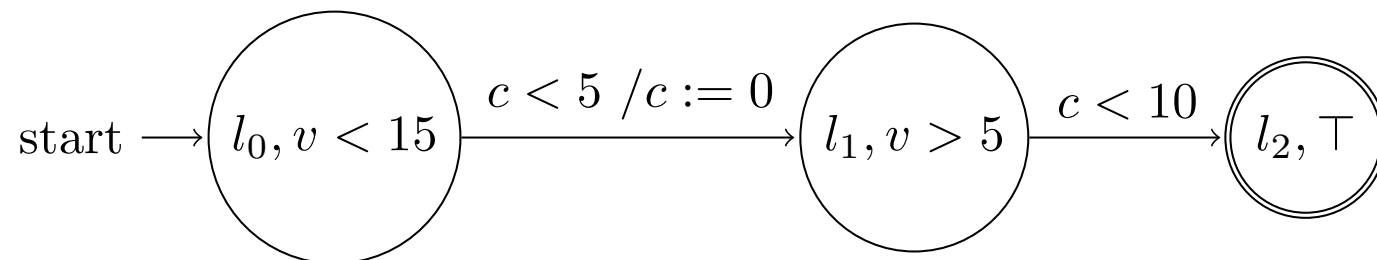
	Boolean	sup-inf	tropical
S	{True/False}	$\mathbb{R} \cup \{\pm\infty\}$	$\mathbb{R} \cup \{+\infty\}$
\oplus	\vee	sup	inf
\otimes	\wedge	inf	+

Local Conclusion: Zone Construction with Weight

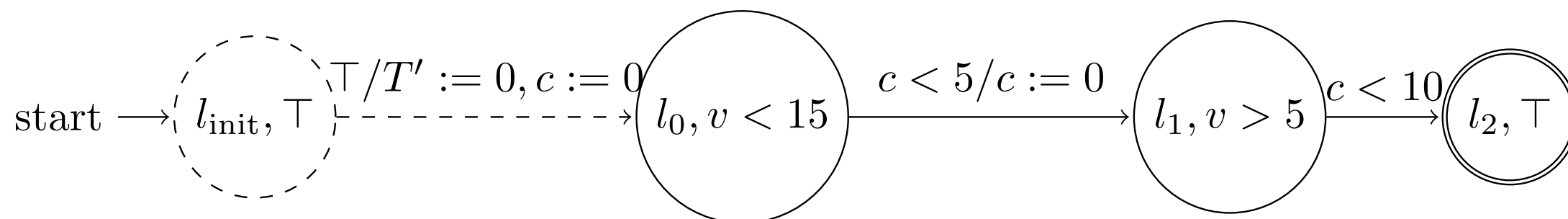
- The construction is basically same as the usual zone construction
- Weights are same as weighted TTS
- The state space is **finite** thanks to **zones** and **finite horizon** of the input signal

Matching Automata for Pattern Matching

[Bakhirkin+, FORMATS'18]



- Add l_{init} to wait for the beginning of the matching
- Add clock variable T' for the beginning of the matching



Outline

- Motivation + Introduction
- Technical Part
 - Timed symbolic weighted automata (TSWA)
 - TSWA: TA with signal constraints + weight function
 - Quantitative monitoring/timed pattern matching algorithm
 - Idea: zone construction with weight
- Experiments

Environment of Experiments

- **Semirings:** sup-inf ($\mathbb{R} \cup \{\pm \infty\}$, sup, inf) and tropical ($\mathbb{R} \cup \{+\infty\}$, inf, +)
- Used 3 original benchmarks (automotive):
 - Inspired by ST-Lib [Kapinski+, SAE Technical Paper'16]

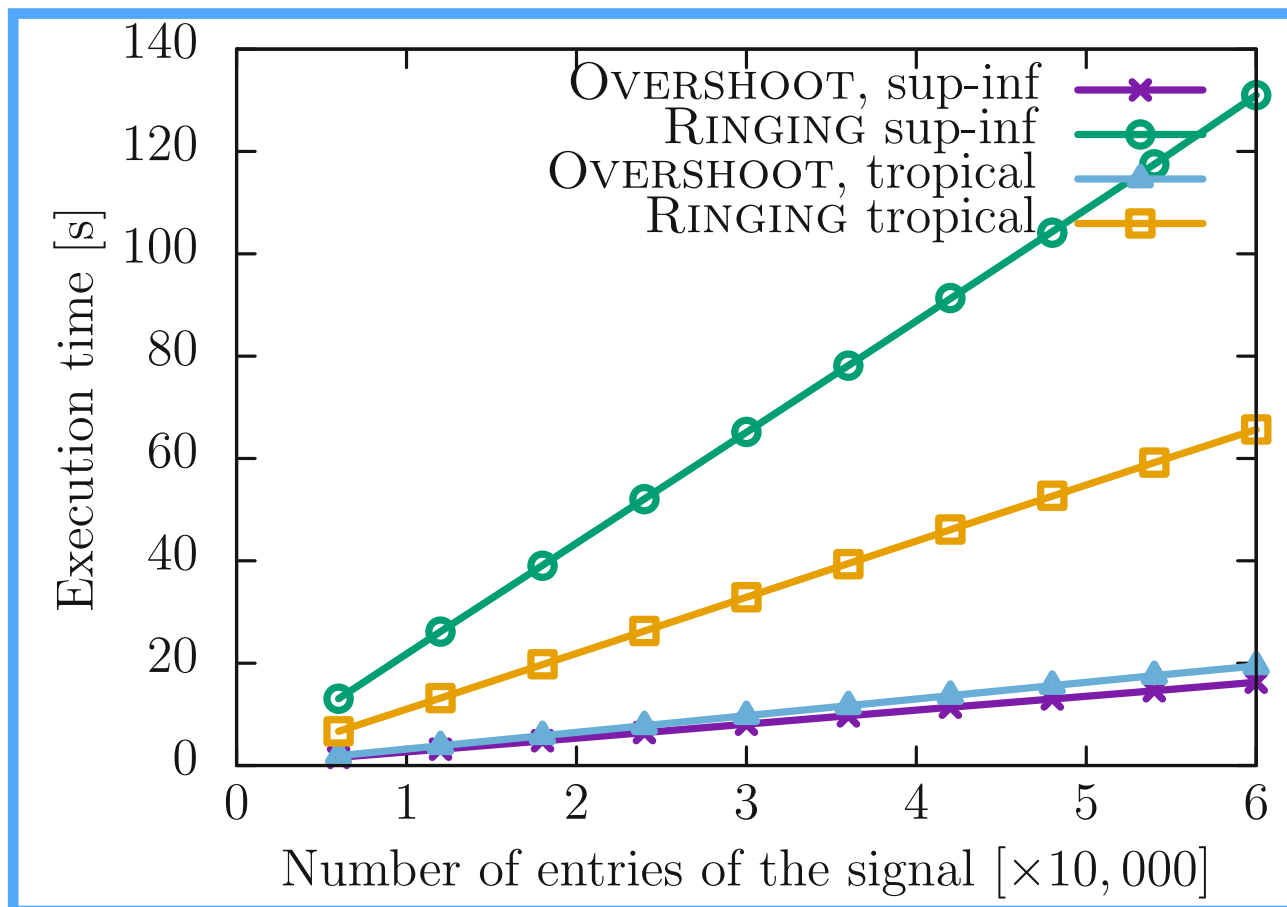
- **Overshoot:** $|v_{\text{ref}} - v|$ gets large after v_{ref} changed
 - Only matches the sub-signals of **length < 150 time units**
- **Ringling:** $v(t) - v(t-10)$ gets positive and negative repeatedly
 - Only matches the sub-signals of **length < 80 time units**

- **Overshoot (unbounded):** $|v_{\text{ref}} - v|$ gets large after v_{ref} changed
 - No such *bounded*

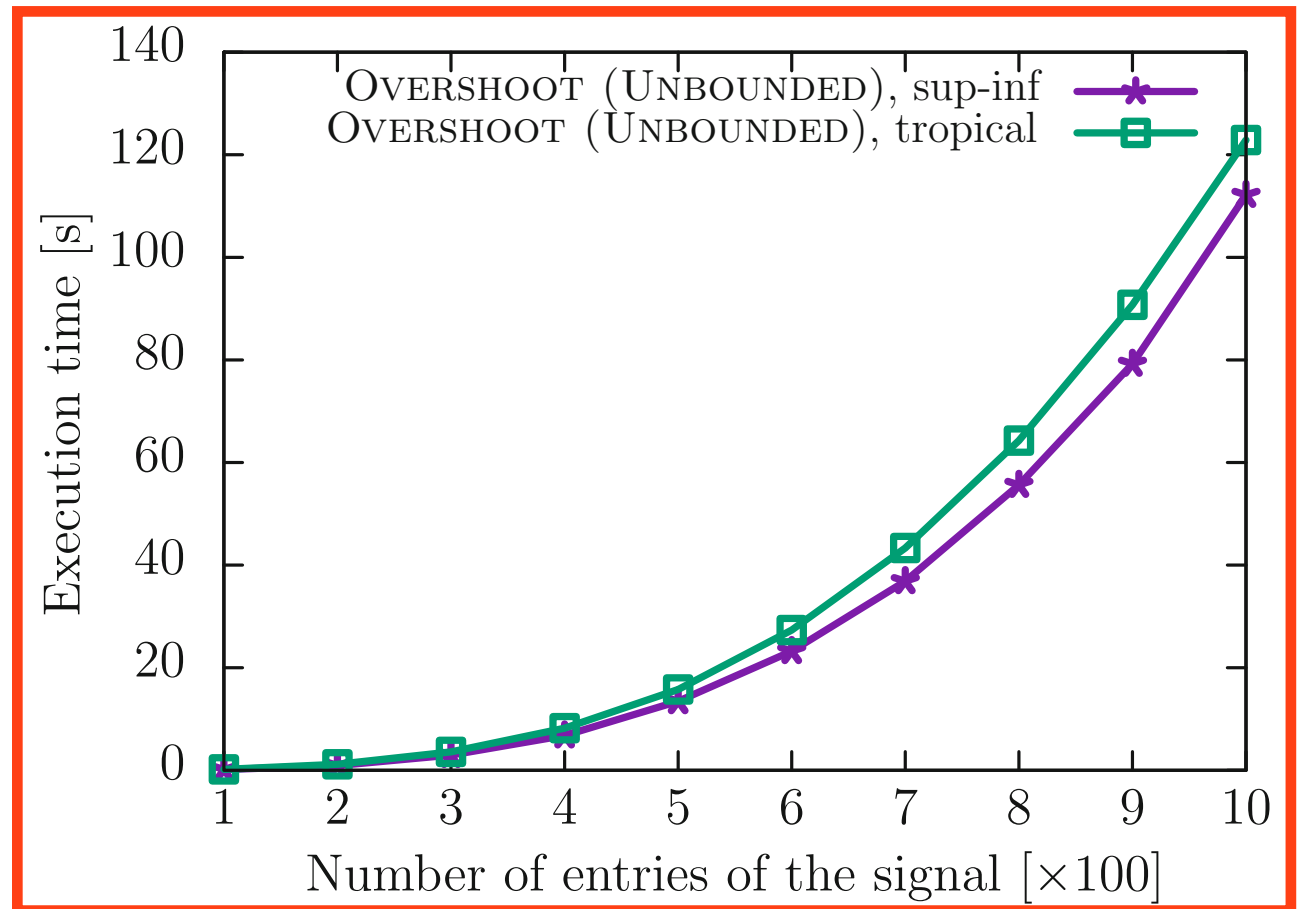
- Amazon EC2 c4.large instance / Ubuntu 18.04 LTS (64 bit)
 - 2.9 GHz Intel Xeon E5-2666 v3, 2 vCPUs, 3.75 GiB RAM

Execution Time

Bounded



Unbounded



- Execution time is **linear** for the bounded spec.
 - 1,000 entries / 1 or 2 sec.
- Execution time explodes for the unbounded spec.

Conclusion

- Introduced timed symbolic weighted automata (**TSWA**)
 - **TSWA**: TA with signal constraints + weight function
- Gave quantitative monitoring/timed pattern matching algorithm
 - **Idea**: zone construction with weight
- Implementation + experiments
 - **scalable** for bounded specifications

Appendix

Example: “Robust” Semantics

Weight Function: minimum distance from the threshold

$$\kappa_r(u, (a_1 a_2 \dots a_m)) = \inf_{i \in \{1, 2, \dots, n\}} \kappa_r(u, (a_i))$$

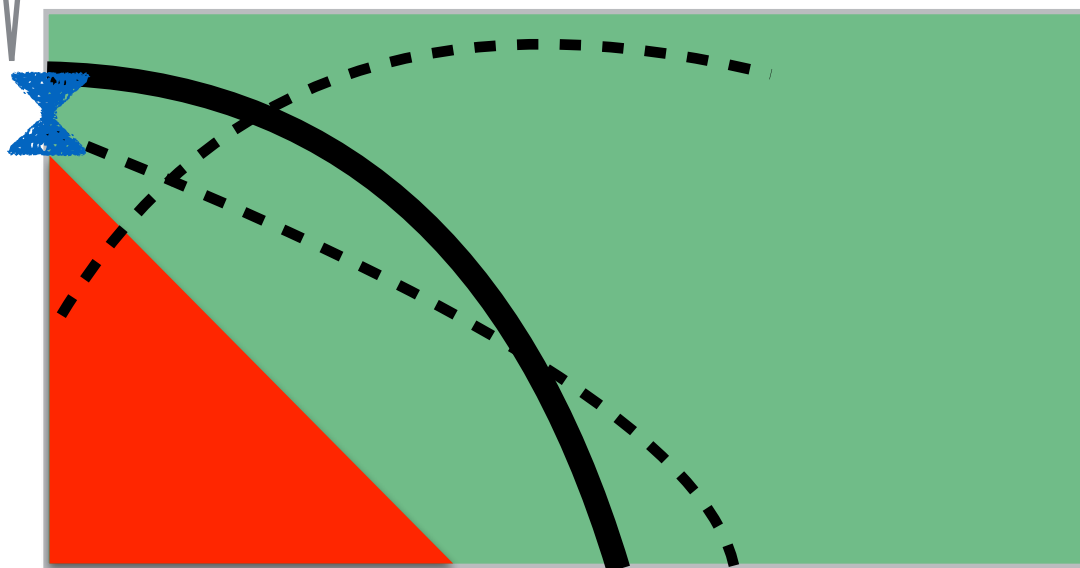
$$\kappa_r\left(\bigwedge_{i=1}^n (x_i \bowtie_i d_i), (a)\right) = \inf_{i \in \{1, 2, \dots, n\}} \kappa_r(x_i \bowtie_i d_i, (a)) \text{ where } \bowtie_i \in \{>, \geq, \leq, <\}$$

$$\kappa_r(x \succ d, (a)) = a(x) - d \quad \text{where } \succ \in \{\geq, >\}$$

$$\kappa_r(x \prec d, (a)) = d - a(x) \quad \text{where } \prec \in \{\leq, <\}$$

Robustness

Semiring: sup-inf semiring



Example: “Robust” Semantics

Weight Function: minimum distance from the threshold

$$\kappa_r(u, (a_1 a_2 \dots a_m)) = \inf_{i \in \{1, 2, \dots, n\}} \kappa_r(u, (a_i))$$

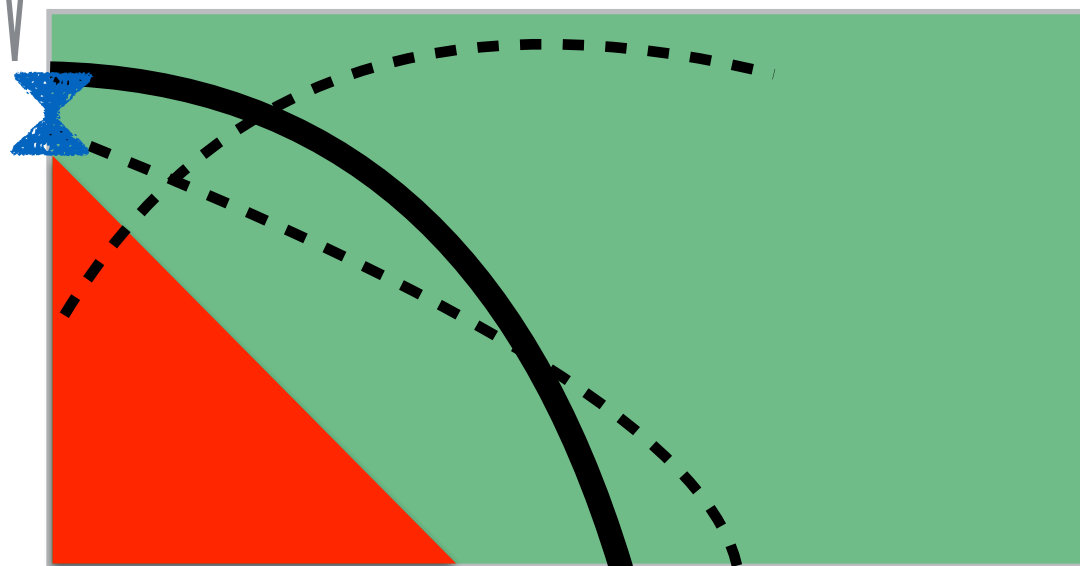
$$\kappa_r\left(\bigwedge_{i=1}^n (x_i \bowtie_i d_i), (a)\right) = \inf_{i \in \{1, 2, \dots, n\}} \kappa_r(x_i \bowtie_i d_i, (a)) \text{ where } \bowtie_i \in \{>, \geq, \leq, <\}$$

$$\kappa_r(x \succ d, (a)) = a(x) - d \quad \text{where } \succ \in \{\geq, >\}$$

$$\kappa_r(x \prec d, (a)) = d - a(x) \quad \text{where } \prec \in \{\leq, <\}$$

Robustness

Semiring: sup-inf semiring

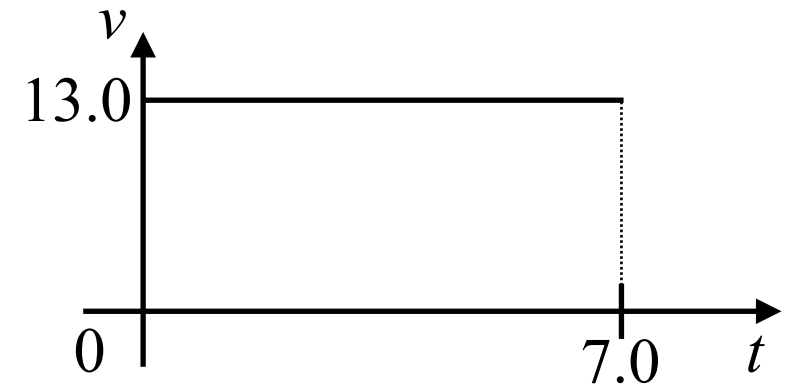
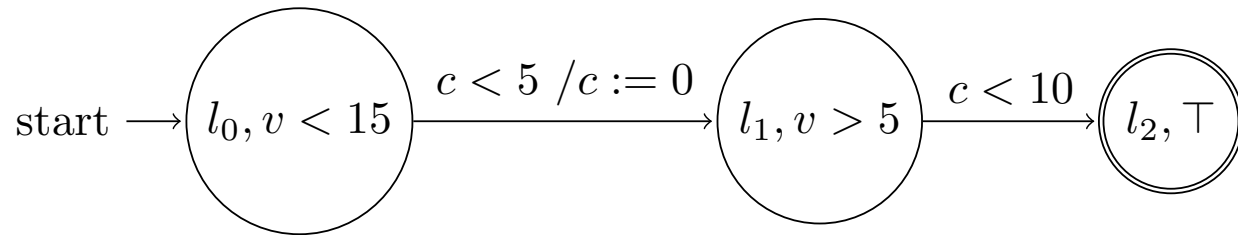


	Boolean	sup-inf	tropical
S	{True/False}	$\mathbb{R} \cup \{\pm\infty\}$	$\mathbb{R} \cup \{+\infty\}$
\oplus	\vee	sup	inf
\otimes	\wedge	inf	+

Insights: Zone Construction with Weight

- The construction is basically same as the usual zone construction
- The state space is **finite** thanks to **zones** and **finite horizon** of the input signal
- The weight is **constant** because the signal is **piecewise-constant**

Comparison of the semiring



$\rightarrow (l_0, c=0, 0, \varepsilon) \dashrightarrow (l_0, c=2, 2, \{v=7\}) \rightarrow (l_1, c=0, 3, \varepsilon) \dashrightarrow (l_1, c=5, 7, \{v=7\}\{v=12\}) \rightarrow (l_2, c=5, 7, \varepsilon)$

$\kappa(v < 15, \{v=13\}) = 2 \quad \otimes \quad \kappa(v > 5, \{v=13\}) = 8$

Sup-inf semiring

$2 \otimes 8 = \inf(2, 8) = 2$

Tropical semiring

$2 \otimes 8 = 2 + 8 = 10$

	Boolean	sup-inf	tropical
S	{True/False}	$\mathbb{R} \cup \{\pm\infty\}$	$\mathbb{R} \cup \{+\infty\}$
\oplus	\vee	sup	inf
\otimes	\wedge	inf	+